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Assessing Fiscal Behavior of Asian Economies to Foreign Capital

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Abstract

The current study has empirically estimated the impact of foreign capital in terms of grants and loans on the fiscal behavior of nine developing countries for Asian region over a period of 1984-2015. For this purpose it developed fiscal response model and used Non-linear Three-Stage Least Squares method for estimation. The results of the study revealed that 84 percent of loans money was used for consumption purpose and merely 16 percent of loans were channeled towards investment purpose that shows that the governments of this region heavily relying on external loans to meet recurrent expenditures. However the study has also an interesting finding that the most of the grant money goes for development purpose with a little leakage into the consumption. And the loans have pro-consumption effect while grants have pro-investment effect. It also found that 67 percent of the tax money is spent on recurrent expenditures while 33 percent is used for developmental purpose. The study proposes that the governments of this region should minimize its reliance on external sources particularly on loans and it should ensure that they are more directed towards developmental purpose. Moreover, Government should follow the policy of self reliance and should increase its tax base and tax net. It also proposes that health and education sector should be given priority.

Keywords-- Asia **,**foreign aid,foreign grants, fiscal response model, Non-linear 3sls,

Introduction

The foreign capital plays a crucial role in the development process of recipient countries. Many of the Asian countries are developing and to make progress and develop, they are highly dependent on foreign assistance in shape of grant and loans. The real fruit of assistance could be reaped only if these inflows are spent on concrete targets and for development purpose. For the donors it is important to ensure that aid has been utilized appropriately. This is also important for recipients as aid loans are usually associated with highly repayment obligations.

With time the circumstances of Asian countries have been changed and their ability to engage in commercial financial market and mobilization of domestic resources has been improved. So to get a better idea regarding the role of external finance on the fiscal behavior a comprehensive analysis of this nature is required.

The current study has modified and extended the fiscal response model which was introduced by Heller (1975). This study concentrates on a couple of research questions that will attempt to explore the impact of foreign aid on fiscal behavior of ten major Asian countries through fiscal response model. The following research questions has been addressed throughout this study

1. Is aid in form of Loans and Grants associated with increase in public development expenditures on health, education?
2. Does aid (grants and loans) have positive impact on tax revenues?
3. Does tax money is spent on capital/ investment expenditures or used for recurrent expenditures?
4. Does aid (loans, grants) have pro-investment or pro-consumption behavior

The aid is further disaggregated into two types, aid loans and aid grants. The difference between aid loans and aid grants is that former are to be paid back while grants money is not returned back at all.

The countries included for this study are Bangladesh, India, Indonesia, Malaysia, Nepal, Pakistan, Philippines, Sri Lanka and Thailand. The main contribution of this study is that it is cross country study while most of the past studies were either country specific (e.g., Franco-Rodriguez, 2000, 2008; McGillivray, 2000, 2009; Bhattarai, 2007; Batten, 2010; Aregbeyen *et al.*, 2014)or focused on African countries (e.g., Heller, 1975; Senbet *et al*., 2009; Aregbeyen *et al.*, 2014). Further it has also tried to capture the impact of foreign capital on domestic taxes, development expenditures, consumption expenditures and investment expenditures separately.

Review of Literature

The first study that used fiscal response model was of Heller (1975) and unfortunately it remained unnoticed in the aid literature for a number of years until Gang and Khan (1991) realized its importance and used this model in his study. So Gang and Khan(1991) study was the second one that assessed the impact of aid on the fiscal behavior of the Indian Economy from 1961-1984. Later, Khan and Hoshino (1992), McGillivray (2000), Franco-Rodriguez (2000), Mavrotas (2002, 2005), Feeny (2006, 2007), Mavrotas and Ouattara (2006,) Ouattara (2006a) have followed Heller (1975) and Gang and Khan (1991) work and made their own contribution to fiscal response literature.

Note that only two studies; Khan and Hoshino (1992) and Ahmed (1996) focused on Asian economies. Khan and Hoshino (1992) made cross- country study on five Asian countries (Pakistan, India, Bangladesh, Malaysia, Sri Lanka) over the period 1955-1976, while Ahmed (1996) made his study on four Asian countries (Pakistan, India, Bangladesh, Philippines) using data from the 1960s to the early 1990s. Rest all studies were either country specific or focused on African economies.

In contrast, this study instead of using single country analysis extend the fiscal response model to ten major Asian economies namely; Bangladesh, India, Indonesia, Malaysia, Nepal, Pakistan Philippines, Sri Lanka, Thailand, and Vietnam over a period of 1984-2015 by employing more recent data. This makes it different from all other past studies and acknowledges its importance. Moreover it has also tried to capture the impact of foreign capital on domestic taxes, development expenditures, consumption expenditures and investment expenditures separately which was not done in past. So this makes this study a significant contribution to the literature

Methodology

The main purpose of this study is to investigate the impact of aid on major fiscal variables such as tax revenue, government consumption expenditures, government socio-economic expenditures/development expenditures, government investment expenditures. The foreign aid is further disintegrated into grants and loans. The study will use fiscal response model and will estimate it by using non-linear three stages least squares (3SLS) technique The main variables of present panel data study include foreign aid, foreign grants, government tax revenues, government consumption expenditures, government socio-economic expenditures. The annual data from 1984-2015 is taken from WDI, IMF Government Financial Statistics (GFS), Global Development Finance (World Bank).

Estimation of Model

This study will employ panel data. Panel data refers to multi-dimensional data frequently involving measurements over time. The government expenditures are broadly divided into two major categories recurrent expenditures and non-recurrent expenditures. Recurrent expenditures are further divided into development expenditures (Gs) and non-development expenditures/consumption expenditures (Gc). Development expenditures include social expenditures such as expenditures made on education and health. While non-development expenditures include government expenditures for military, security purposes and wages made to the civil servants. While the government revenue includes domestic revenues/ tax income (T), foreign inflow of capital is in form of grants (A_1) and loans (A_2).

The utility function of the public sector can be represented as follows:

Utility = f (Govt. Capital Expenditures, Govt. development expenditures, Govt. consumption expenditure, Domestic Revenues, Domestic Borrowing, Grants, Loans)

$$U= f(G_I ,G_S, G_c,T, B_D, A_G, A_L) \quad [1]$$

Hrere

G_I = Govt. capital/investment expenditures

G_S = Govt. Development Expenditures (health, Education)

G_c = Govt. consumption expenditure

T = Domestic Revenues/Tax

B_D = Domestic borrowing

A_G = Grant aid

A_L = Loan aid

This utility function can be maximized subject to the budget constraints

$$Gs+ Gc= T\rho_1 + AG\rho_2 + AL\ \rho_3 \quad [2]$$

$$Gi = B+T(1-\rho_1)+(1-\rho_1)AG+(1-\rho_1)\ AL \quad [3]$$

Hence ρ_1 , ρ_2 and ρ_3 are parameters that measure the proportion of domestic revenues, grants and loans allocated to government recurrent expenditures while (1-ρ_1) measures the proportion of domestic revenues, grants and loans allocated for public investment expenditures.

As there are two budget constraints so the utility function would be specified by introducing two Lagrange multipliers λ_1 and λ_2.

$$U=\beta_0 +\beta_1 (G_I -G_I^*) -\frac{\beta 2}{2} (G_I -G_I^*)^2- \beta_3 (T - T^*) -\frac{\beta 4}{2}(T - T^*)^2 +\beta_5 (G_C -G_C^*) -\frac{\beta 6}{2}(G_C -G_C^*)^2+\beta_7 (G_S -G_S^*) -\frac{\beta 8}{2}(G_S -G_S^*)^2- \beta_9 (B_D - B_D^*) -\frac{\beta 10}{2}(B_D - B_D^*)^2 +\beta_{11} (A_G - A_G^*) --\frac{\beta 12}{2}(A_G - A_G^*)^2+\beta_{13} (A_L - A_L^*) -\frac{\beta 14}{2}(A_L - A_L^*)^2+\lambda_1\{G_I -B_D -(1-\rho_1) T -(1-\rho_2) A_L-(1-\rho_3) A_G\}+ \lambda_2 \{G_C +G_S -\rho_1T- \rho_2 A_L- \rho_3 A_G\}$$

For optimization of this function, we will take first order derivate of the lagrangian function with respect to all the choice variables and two langrangian multipliers i.e. λ_1 and λ_2.

The variables with asterisk (*) sign are the target values of these variables

Construction of Target Variables

For the estimation of the above model target values of the choice variables are required and hence they are generated by following the footsteps of Heller (1975) and Gang and Khan (1991, 1996). These target values has been estimated because there was no published source which provide target data.

- Target Value of Public Investment (G_I*)
- Target Value of Domestic Revenues (**T***)
- Target Value of Public Consumption (**Gc***)
- Target Value of development Exp (**Gs***)
- Target Value of Aid Grants(**A_G***)
- Target Value of Aid Loans (**A_L***)
- Target Value of Domestic Borrowing (**B_D***)

$$\mathbf{G_I^* =\beta_{15}+\beta_{16}\ Y_{t-1}+\beta_{17}Ip+\beta_{18}G_{It-1}(4)}$$

(Target Govt. capital expenditures/Public investment)*= β_{15}+ β_{16} (GDP) $_{t-1}$+ β_{17} (Private Investment) + β_{18}(Lagged Govt. capital expenditures/Public Investment)

Equation (4) estimates the target value of public investment (G_I*) using "accelerator principle". It also establishes the relationship between private and public investment.

$$\mathbf{T^* = \beta_{19}+\beta_{20} Y+\beta_{21} M+\beta_{22}T_{t-1}} \quad \mathbf{(5)}$$

(Target Domestic Revenues)*= β_{19}+ β_{20} (GDP/Economic Activity) + β_{21}(Imports) + β_{22}(Lagged Domestic Revenues)

Equation (5) measures the target value of the domestic revenue (T*) which depends on economic activity/ GDP, imports and one period lagged value of domestic revenue.

$$\mathbf{Gc^* = \beta_{23}+\beta_{24} Gc_{t-1}+\beta_{25}Y} \quad \mathbf{(6)}$$

(Target Public Consumption)*=β_{23}+β_{24}(lagged Public consumption) +β_{25} (GDP/Economic Activity)

Equation (6) estimates the target value of public consumption expenditures (**Gc***) which depends on economic activity/GDP and one period lagged value of the public consumption expenditures.

$$\mathbf{Gs^* = \beta_{26}+\beta_{27}P+\beta_{28}Y +\beta_{29}Gs_{t-1}} \quad \mathbf{(7)}$$

(Target Socio-economic Exp)= β_{26}+β_{27} (Population) +β_{28} (GDP) +β_{29}(lagged Socio-economic Exp)

Equation (7) measures the target value of socio-economic expenditures (**Gs)** which depends upon population, output/ GDP and one period lagged value of socio- economic expenditures.

$$\mathbf{A_G = \beta_{30}+\beta_{31}A_{Gt-1}} \quad \mathbf{(8)}$$

Aid Grants = β_{30}+β_{31} (lagged Aid Grants)

Equation (8) assumes that aid grant depends upon its lagged value.

$$\mathbf{A_L = \beta_{32} +\beta_{33}A_{Lt-1}} \quad \mathbf{(9)}$$

Aid Loans=β_{32} +β_{33}(lagged Aid Loans)

Equation (9) assumes that aid loan depends on its lagged value

$$\mathbf{B_D^*=0} \quad \mathbf{(10)}$$

B_D = Domestic borrowing

Equation (10) is assumed to be zero. For this we followed Heller (1975), Gang and Khan (1991), Khan and Hoshino (1992), and Otim (1996) who assumed that there is no behavioral equation for domestic borrowing as it is considered to be last option for government finance and it also creates high inflation and crowding out.

Since the expenditure side should be equal to finance side of the equation

$$\mathbf{G_I + G_c+ G_S= T+ B_D+ A_G+ A_L} \quad \mathbf{[11]}$$

The government expenditures are categorized into Govt. capital expenditures and government recurrent expenditures. Govt. recurrent expenditures further subdivided into expenditures for socio-economic ends (which include expenditures on health and education) and Govt. consumption expenditures (military expenditures and wages of civil servants).While receipts includes domestic revenues, foreign aid in form of ODA grants, foreign aid in form of ODA loans and revenue generated from domestic borrowing. It is worth mentioning here that domestic borrowing is the last option to finance government expenditures because there is high cost attached to it in terms of high rate of inflation, crowding out of the investment (Heller, 1975).And the government of developing countries avoid financing expenditures from domestic borrowing and rely on the domestic revenue and foreign capital inflow to finance both investment and consumption expenditures (Heller, 1975).

Structural Equations

By applying the first order condition and after some mathematical calculations the following structural equations have been estimated

$$Gs=\beta_0-(1-\beta_1)Gc^*-\beta_1 Gs^*+(1-\beta_1)\rho_1 T+(1-\beta_1)\rho_2 AG+(1-\beta_1)\rho_3 AL+e_1 \quad [A]$$

$$Gc=\beta_0+(1-\beta_1)Gc^*-\beta_1 Gs^*+\beta_1\rho_1 T+\beta_1\rho_2 AG+\beta_1\rho_3 AL+e_2 \quad [B]$$

$$T=\beta_8+\rho_1\beta_9(Gc-Gc^*)+\beta_{10}T^*+\beta_{11}(1-\rho_1)GI-\beta_{11}(1-\rho_1)(1-\rho_2)AG-\beta_{11}(1-\rho_1)(1-\rho_3)AL+e_3 \quad [C]$$

$$GI=\beta_{12}+\beta_{13}IG^*+(1-\rho_1)(1-\beta_{13})T+(1-\rho_2)(1-\beta_{13})AG+(1-\rho_3)(1-\beta_{13})AL+e_4 \quad [D]$$

The estimates of the structural parameters (βs) show only the partial effect but not the total effect

As the system involve cross parameter restrictions and these equations are non-linear with respect to ρ and β parameters. The relative inefficiency of limited information estimation technique and considering the cross parameter restriction, the study has opted a full information technique named Three Stages-least squares (3SLS).This technique is superior to Two-Stage Least Square (2SLS) as it apply generalized least square in the third stage to the residuals generated from second stage regression of Two-Stage Least Square (2SLS) [Mukherjee et al 1998].

Results

The non-linear 3SLS estimation technique is used to estimate the model. In estimation procedure, the order condition for identification is equivalent in saying the instrumental variables should be as many as the coefficient of the equation. So the estimation will yield estimates of ρ_1, ρ2, ρ3, β_0, β_1 and β_8 through β_{13}. The estimates of the structural equations can be calculated through these estimated parameters. The estimates of structural equations describe the total effect of change in the dependent variable/regressand to the change in one of the independent variable/regressor

Table 1 :Estimates of Structural Equation Parameters (Grants & Loans) Asia

Parameter	Coefficient	Standard Error	t- Statistic	P-Value
ρ_1	-0.3291**	0.1444	-2.2791	0.0219
ρ_2	0.3682**	0.1672	2.2026	0.0281
ρ_3	0.8417***	0.3079	2.7336	0.0065
β_0	-8.6286**	3.7368	-2.3091	0.0214
β_1	0.9260***	0.0299	30.9698	0.0000
β_8	28.4028***	3.6171	7.8524	0.0000
β_9	0.1992***	0.0704	2.8291	0.0049
β_{10}	0.0882**	0.0432	2.0402	0.0419
β_{11}	-0.0144**	0.0624	-2.3115	0.0212
β_{12}	-17.9386***	6.0178	-2.9809	0.0030
β_{13}	-0.0078***	0.00053	-14.7169	0.0000

Note: Here *** significant at 1 percent while ** significant at 5 percent

The parameters ρ_1, ρ_2 and ρ_3 are parameters of budget constraint [1]. Budget constraint [1] explains that Gs+ Gc= $T\rho_1 + AG\rho_2 + AL\ \rho_3$ that recurrent expenditures are met with tax money, grants and loans. So, the parameter ρ_1 measures the proportion of tax allocated for recurrent expenditures, ρ2 measures the proportion of grants allocated for recurrent expenditures and ρ_3 measures the proportion of loans allocated for recurrent expenditures. Here ρ_1 = -0.3291 which shows that 33 percent of tax money is pulled out and used for non-recurrent/capital expenditures/investment expenditures while rest 67 percent tax money is spent on recurrent expenditures. This result is consistent with Otim (1996) who found out that ρ_1 is negative and -0.371 for low income south Asian countries.

The parameter ρ_2= 0.3682 is positive and statistically significant at 5 percent it shows that 36 percent grant money is used for recurrent expenditures (consumption purpose) while 63 percent is used for investment purpose. These results are in line with the results of Khan and Hoshino (1992) where ρ_2=0.48, Otim (1996) where ρ_2= 0.344, Franco-Rodriguez *et al* (1998) who found out that ρ_2= 0.51. This result is consistent with Gang and Khan (1991) results that shows that grants goes into development project with no leakage into the consumption.

The third parameter ρ_3 = 0.8417 is positive and statistically significant at 5 percent and shows that 84 percents of loan money is used for recurrent expenditures (consumption purpose) while rest 16 percent is used for investment purpose or public capital expenditures. These results are consistent with Gang and Khan (1996) where ρ_3 = 0.9153 and Franco-Rpdroqiez et al., (1998) where ρ_3 = 0.54. So governments of these regions are heavily relying on external borrowing.

Table 2: Summary of the results of the past studies on Fiscal Behavior

	ρ_1	ρ_2	ρ_3
Heller (1975)	0.83	0.38	-0.39
Gang and Khan(1991)	1.08	-0.79	-0.03
Gang and Khan(1996)	0.46	0.83	0.92
Otim(1996)	-0.37	0.34	0.19
Franco-Rodriguez et al., (1998)	0.85	0.51	0.54
Senbet et al., (2007)	0.49	0.86	0.42

To check the total impact of aid grants and aid loans the following estimation is carried out

Table 3: Estimates of Total Impact of Grants (A_G) and Loans (A_L) -Asia

Impact	Mechanism	Estimate
A_G on T (Impact of Grants on Tax revenues)	$-\beta_{11}(1-\rho_1)(1-\rho_2)$	0.0121
A_G on G_c (Impact of Grants on Public Consumption Expenditures/ Civil Consumption)	$\rho_2 \beta_1$	0.3409
A_G on G_s (Impact of Grants on Socio-economic Expenditures/ Expenditures on Health, Education)	$\rho_2(1-\beta_1)$	0.0272
A_G on I_g (Impact of Grants on Public Investment)	$(1-\beta_{13})(1-\rho_2)$	0.6367
A_L on T (Impact of Loans on Tax revenues)	$-\beta_{11}(1-\rho_1)(1-\rho_3)$	0.0030
A_L on G_c (Impact of Loans on Public Consumption Expenditures/ Civil Consumption)	$\rho_3\beta_1$	0.7794
A_L on G_s (Impact of Loans on Socio-economic Expenditures/Expenditures on Health, Education)	$\rho_3(1-\beta_1)$	0.0622
A_L on I_g (Impact of Loans on Public Investment)	$(1-\beta_{13})(1-\rho_3)$	0.1595

The above table describes the total impact of aid grants (A_G) and Aid Loans (A_L) on fiscal variables such as domestic revenues (T), Public Consumption Expenditures, Development Expenditures Public Investment Expenditure (I_G).

The results show that aid grants and aid loans have positive but very negligible impact on the tax revenues. Around 2 percent of the grant money is spent for socio-economic expenditures (health and education) while merely 6 percents of the loan money is spent for socio-economic purpose (health and education) in this region over the study period. It is also evident that 77 percent of the loan money is used for civil consumption/ non-development purpose and only 15 percent of the amount is allocated for public investment/capital expenditures. So, the result revealed that grants have less pro-consumption effect in comparison to loans. In other words grants were more spent for capital expenditure/investment purpose.

Conclusion and Policy Implications

This paper tries to analyse the fiscal response of public sector of the Asian region in presence of external financial assistance. The external financial assistance is further disintegrated into aid grant and aid loans. There are four major findings of this study are as follows. First, the study results show that a large portion of tax money (68 percent) is channeled towards consumption purpose while only a smaller portion of taxes (33 percent) is used for investment purpose. It is proposed that the governments of this region should keep a check on its tax revenues and should ensure that they are channel towards capital expenditures and used for development purpose.

Second, the results show that loans have pro-consumption effect and 84 percent of loan money is used for consumption purpose whereas only 16 percent of loan money is used for investment purpose. So, the governments of this region is heavily relying on external borrowing and using it irresponsibly which results in heavy debts payments and liabilities. It is suggested that reliance on external sources should be minimized and governments should focus to increase tax revenues by broadening tax base and tax net.

Third, the study has also an interesting finding that the grant money go to development purpose with a smaller leakage into the consumption. While it is also clear that grants have less pro-consumption effect in comparison to loans. In other words grants were more spent for capital expenditure/investment purpose.

Fourth, the results reveal that grants and loans have positive but negligible impact on tax revenues. However, it is suggests that government should spend adopt the policy of self reliance and should give priority to the socio-economic development expenditures especially on health and education, this will in return result in reduction in inequality and poverty.

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Appendix

As there are two budget constraints so the utility function would be specified by introducing two Lagrange multipliers λ_1 and λ_2.

$$U=\beta_0+\beta_1 (G_I - G_I^*) - \frac{\beta 2}{2}(G_I - G_I^*)^2 - \beta_3 (T - T^*) - \frac{\beta 4}{2}(T - T^*)^2 + \beta_5 (G_C - G_C^*) - \frac{\beta 6}{2}(G_C - G_C^*)^2 + \beta_7 (G_S - G_S^*) - \frac{\beta 8}{2}(G_S - G_S^*)^2 - \beta_9 (B_D - B_D^*) - \frac{\beta 10}{2}(B_D - B_D^*)^2 + \beta_{11} (A_G - A_G^*) -- \frac{\beta 12}{2}(A_G - A_G^*)^2 + \beta_{13} (A_L - A_L^*) - \frac{\beta 14}{2}(A_L - A_L^*)^2 + \lambda_1\{G_I - B_D - (1-\rho_1) T - (1-\rho_2) A_L - (1-\rho_3) A_G\} + \lambda_2 \{G_C + G_S - \rho_1 T - \rho_2 A_L - \rho_3 A_G\}$$

For optimization it is required to take the first order derivate of the lagrangian function with respect to the all choice variables and two lagrangian multiplier λ_1 and λ_2.

The first order conditions are as follow

$$\frac{dU}{dGI} = \beta 1 - \beta 2(GI - GI*) + \lambda 1 = 0 \qquad [1]$$

$$\frac{dU}{dGc} = \beta 5 - \beta 6(Gc - Gc*) + \lambda 2 = 0 \qquad [2]$$

$$\frac{dU}{dGs} = \beta 7 - \beta 8(Gs - Gs*) + \lambda 2 = 0 \qquad [3]$$

$$\frac{dU}{dT} = -\beta 3 - \beta 4(T - T*) - \lambda 1\,(1 - \rho 1) - \lambda 2\,\rho 1 = 0 \qquad [4]$$

$$\frac{dU}{dBD} = -\beta 9 - \beta 10(BD - BD*) - \lambda 1 = 0 \qquad [5]$$

$$\frac{dU}{dAG} = \beta 11 - \beta 12(AG - AG*) - \lambda 1\,(1 - \rho 2\,) - \lambda 2\,\rho 2 = 0 \qquad [6]$$

$$\frac{dU}{dAL} = \beta 13 - \beta 14(AL - AL*) - \lambda 1\,(1 - \rho 3) - \lambda 2\,\rho 3 = 0 \qquad [7]$$

$$\frac{dU}{d\lambda 1} = GI - BD - (1 - \rho 1)T - (1 - \rho 2)AG - (1 - \rho 3)AL = 0 \qquad [8]$$

$$\frac{dU}{d\lambda 2} = Gs + Gc - \rho 1\,T - \rho 2\,AL - \rho 2\,AG = 0 \qquad [9]$$

The first order conditions [1] to[9] are solved together to solve the structural equations

From equation [9]

$Gs = \rho 1\,T + \rho 2\,AL + \rho 2\,AG - Gc$ [9a]

Equation [2] and [3] are solved for $\lambda 2$

$\lambda 2 = \beta 6\,Gc - \beta 6\,Gc * - \beta 5$ [2a]

$\lambda 2 = \beta 8\,Gs - \beta 8\,Gs * - \beta 7$ [3a]

From Equation [2a] and [3a]

$$\lambda 2 = \lambda 2$$

$$Gc = \frac{\beta 8\,Gs + \beta 6\,Gc* - \beta 8Gs* + \beta 5 - \beta 7}{\beta 6} \qquad [10]$$

$$Gs = \frac{\beta 6\,Gc - \beta 6\,Gc * + \beta 8Gs * + \beta 7 - \beta 5}{\beta 8} \qquad [11]$$

Substituting equation [10] into [9a]

$$Gs = \rho 1\,T + \rho 2\,AL + \rho 3\,AG - Gc \qquad [9a]$$

$$Gs = \rho 1\,T + \rho 2\,AG - \rho 3\,AL - \frac{\beta 8\,Gs + \beta 6\,Gc * - \beta 8Gs * + \beta 5 - \beta 7}{\beta 6}$$

$$Gs + \frac{\beta 8\,Gs}{\beta 6} = \frac{\beta 6\,\rho 1\,T + \beta 6\,\rho 2\,AG + \beta 6\,\rho 3\,AL - \beta 6Gc * - \beta 8Gs * + \beta 5 - \beta 7}{\beta 6}$$

$$\frac{\beta 6\,Gs + \beta 8\,Gs}{\beta 6} = \frac{\beta 6\,\rho 1\,T + \beta 6\,\rho 2\,AG + \beta 6\,\rho 3\,AL - \beta 6Gc * - \beta 8Gs * + \beta 5 - \beta 7}{\beta 6}$$

$$\frac{Gs(\beta 6 + \beta 8)}{\beta 6} = \frac{\beta 6\,\rho 1\,T + \beta 6\,\rho 2\,AG + \beta 6\,\rho 3\,AL - \beta 6Gc * - \beta 8Gs * + \beta 5 - \beta 7}{\beta 6}$$

$$Gs = \left[\left(\frac{\beta 7 - \beta 5}{\beta 6}\right)\left(\frac{\beta 6}{\beta 6 + \beta 8}\right)\right] + \left[\left(\frac{\beta 6}{\beta 6 + \beta 8}\right)\left(\frac{\beta 6\,\rho 1\,T}{\beta 6}\right)\right] + \left[\left(\frac{\beta 6}{\beta 6 + \beta 8}\right)\left(\frac{\beta 6\rho 2\,AG}{\beta 6}\right)\right] + \left[\left(\frac{\beta 6}{\beta 6 + \beta 8}\right)\left(\frac{\beta 6\,\rho 3\,AL}{\beta 6}\right)\right] - \left[\left(\frac{\beta 6}{\beta 6 + \beta 8}\right)\left(\frac{\beta 6\,Gc *}{\beta 6}\right)\right] - \left[\left(\frac{\beta 6}{\beta 6 + \beta 8}\right)\left(\frac{\beta 8\,Gs *}{\beta 6}\right)\right]$$

$$Gs = \left[\left(\frac{\beta 7 - \beta 5}{\beta 6 + \beta 8}\right)\left(\frac{\beta 6}{\beta 6 + \beta 8}\right)\right]\rho 1\,T + \left[\left(\frac{\beta 6}{\beta 6 + \beta 8}\right)\rho 2\,AG + \left(\frac{\beta 6}{\beta 6 + \beta 8}\right)\rho 3\,AL\right] - \left[\left(\frac{\beta 6}{\beta 6 + \beta 8}\right)Gc *\right] - \left[\left(\frac{\beta 8}{\beta 6 + \beta 8}\right)Gs *\right]$$

$$Gs = \beta 0 - (1 - \beta 1)Gc * - \beta 1Gs * + (1 - \beta 1)\rho 1\,T + (1 - \beta 1)\rho 2\,AG + (1 - \beta 1)\rho 3\,AL \qquad [A]$$

Where

$\beta 0 = \frac{(\alpha 7 + \alpha 5)}{(\alpha 6 + \alpha 8)}$, $\beta 1 = \frac{\alpha 8}{(\alpha 6 + \alpha 8)}$, $1 - \beta 1 = \frac{\alpha 6}{(\alpha 6 + \alpha 8)}$

From Equation [9a]

$$Gs = \rho 1\ T + \rho 2\ AL + \rho 2\ AG - Gc$$

Substituting equation [10] into [9a]

$$Gc = \rho 1\ T + \rho 2\ AG - \rho 3\ AL - [\beta 0 - (1-\beta 1)Gc* + \beta 1 Gs* + (1-\beta 1)\rho 1\ T + (1-\beta 1)\rho 2\ AG + (1-\beta 1)\rho 3\ AL$$

$$Gc = \rho 1\ T + \rho 2\ AG - \rho 3\ AL - \beta 0 + (1-\beta 1)Gc* - \beta 1 Gs* - (1-\beta 1)\rho 1\ T - (1-\beta 1)\rho 2\ AG - (1-\beta 1)\rho 3\ AL$$

$$Gc = -\beta 0 + \rho 1\ T - (1-\beta 1)\rho 1\ T + \rho 2\ AG - (1-\beta 1)\rho 2\ AG + \rho 3\ AL - (1-\beta 1)\rho 3\ AL + (1-\beta 1)Gc* - \beta 1 Gs*$$

$$Gc = -\beta 0 + \rho 1\ T - \rho 1\ T + \beta 1 \rho 1\ T + \rho 2\ AG - \rho 2\ AG + \beta 1 \rho 2\ AG + \rho 3\ AL - \rho 3\ AL + \beta 1 \rho 3\ AL + (1-\beta 1)Gc* - \beta 1 Gs*$$

$$Gc = -\beta 0 + (1-\beta 1)Gc* - \beta 1 Gs* + \beta 1 \rho 1\ T + \beta 1 \rho 2\ AG + \rho 3 \beta 1 AL \quad [B]$$

We solve Equation [1] and [5] for $\lambda 1$

$$\lambda 1 = \beta 1 - \beta 2(GI - GI*)\ [1a]$$

$$\lambda 1 = -\beta 9 - \beta 10(BD - BD*)\ \ [5a]$$

Solving Equation [1a] and [5a] for $\lambda 1$ and assuming B*= 0

$$B = \frac{(\beta 1 - \beta 9) + \beta 2 GI* - \beta 2 GI}{\beta 10} \quad [14]$$

From equation [8]

$GI = BD + (1-\rho 1)T + (1-\rho 2)AG + (1-\rho 3)AL = 0$ [8a]

Substituting equation [14] into [8a]

$$GI = \frac{(\beta 1 - \beta 9)}{\beta 10 + \beta 2} + \frac{\beta 2}{\beta 10 + \beta 2} IG* + \frac{\beta 10}{\beta 10 + \beta 2}\{(1-\rho 1)T + (1-\rho 2)AG + (1-\rho 3)AL\}\ [C]$$

From Equation [8]

$GI - BD - (1-\rho 1)T - (1-\rho 2)AG - (1-\rho 3)AL = 0$ [8]

$$(1-\rho 1)T = \{GI - (1-\rho 2)AG - (1-\rho 3)AL - BD\}$$

$T = \frac{\{GI - (1-\rho 2)AG - (1-\rho 3)AL - BD\}}{(1-\rho 1)}$ [8b]

From equation [5], assume B*=0

$$-\beta 9 - \beta 10(BD - BD*) - \lambda 1 = 0\ [5]$$

$$BD = \frac{(\beta 9 + \lambda 1)}{\beta 10} \quad [5a]$$

From equation [2]

$$\beta 5 - \beta 6(Gc - Gc*) + \lambda 2 = 0 \quad [2]$$

$$\lambda 2 = \beta 6(Gc - Gc*) - \beta 5 \quad [2a]$$

Substitution equation [2a] into [4]

$$-\beta 3 - \beta 4(T - T*) - \lambda 1\ (1 - \rho 1\ ) - \lambda 2\ \rho 1\ = 0\ \ [4]$$
$$-\beta 3 - \beta 4T + \beta 4T* - \lambda 1\ (1 - \rho 1\ ) - \lambda 2\ \rho 1\ = 0$$
$$\lambda 1 = \frac{-\beta 3 - \beta 4T + \beta 4T* - \lambda 2\ \rho 1}{(1 - \rho 1\ )}$$

$\lambda 1 = \frac{-\beta 3 - \beta 4T + \beta 4T* - \rho 1\beta 6(Gc - Gc*) - \beta 5\ \rho 1}{(1-\rho 1\ )}$ [15]

Substituting equation [15] into [5a]

$BD = \frac{(\beta 9+\lambda 1)}{\beta 10}$ [5a]

$BD = \frac{-\beta 9(1-\rho 1\ )+\beta 3+\beta 4T-\beta 4T*+\rho 1\beta 6(Gc-Gc*)-\beta 5\ \rho 1}{\beta 10(1-\rho 1\ )}$ [16]

Substituting equation [16] into [8b]

$$T = \frac{\{GI - (1 - \rho 2)AG - (1 - \rho 3)AL - BD\}}{(1 - \rho 1)}\ [8b]$$

$T = \frac{\alpha 2}{\alpha 3} + \frac{\rho 1\beta 6}{\alpha 3}(Gc - Gc^*) + \beta_4/\alpha_3\ T^* + \beta_{10}/\alpha_3\{(1-\rho_1)(GI - (1-\rho_2)AG - (1-\rho_3)AL)\}$

Where $\alpha 2 = \rho 1\ \beta_5 - \beta_3 + \beta_9 (1-\rho_1)$

$\alpha 3 = \beta_4 + \beta_{10} (1-\rho_1)^2$

From Equation [8], we also have

$\frac{\{GI-BD-(1-\rho 1)T-(1-\rho 3)AL\}}{(1-\rho 2)} = AG$ [18]

Substituting equation [3a] into [6] and solving for λ1

$\lambda 2 = \beta 8\ Gs - \beta 8\ Gs* - \beta 7$ [3a]

$\beta 11 - \beta 12(AG - AG*) - \lambda 1\ (1 - \rho 2\ ) - \lambda 2\ \rho 2\ = 0$ [6]

$\lambda 1 = \frac{\beta 11-\beta 12(AG-AG*)-\rho 2\ \beta 8\ (Gs-Gs*)+\ \rho 2\beta 7}{(1-\rho 2)}$ [19]

Substituting equation [19] into [5a]

$-\beta 9 - \beta 10(BD - BD*) - \lambda 1\ = 0$ [5]

$BD = \frac{-\beta 9-\beta 11+\beta 12(AG-AG*)-\rho 2\ \beta 8\ (Gs-Gs*)+\ \rho 2\beta 7}{\beta 10(1-\rho 2)}$ [20]

Substituting equation [20] into [18] and simplifying

$AG = \frac{\{GI-BD-(1-\rho 1)T-(1-\rho 3)AL\}}{(1-\rho 2)}$ [18]

$T = \beta_8 + \rho_1\ \beta_9\ (Gc-Gc^*) + \beta_{10}T^* + \beta_{11}(1-\rho_1)\ GI - \beta_{11}(1-\rho_1)\ (1-\rho_2)\ AG - \beta_{11}(1-\rho_1)\ (1-\rho_3)AL$ [D]

$AG = \alpha 4/\alpha 5 + \frac{\rho 2\beta 8}{\alpha 5(Gs-Gs*)} + \frac{\beta 14}{\alpha 5}\ AG* + \beta 10/\alpha 5\{(1 - \rho 2)[GI - (1 - \rho 1)T\text{-}(1 - \rho 3)AL]\} + \epsilon 5$

[E]

Where

$\alpha_4 = \rho 2\beta 7 + \beta 9 + \beta 11$

$\alpha_5 = \beta 12 + \beta 10((1-\rho 2)^2$

Similarly using a parallel argument made for AG, It is straightforward to show that

$AL = \alpha 6/\alpha 7 + \frac{\rho 3\beta 8}{\alpha 7(Gs - Gs*)} + \frac{\beta 14}{\alpha 5} AG * + \beta 10/\alpha 7\{(1-\rho 3)[GI - (1-\rho 1)T\text{-}(1-\rho 3)AL]\} + \epsilon 6$ [F]

$\alpha_6 = \rho 3\beta 7 + \beta 9 + \beta 13$

$\alpha 7 = \beta 14 + \beta 10(1-\rho 3)^2$

Therefore the structural equations to be estimated are [A], [B],[C],[D].

Assuming B*=0, the following behavioral equations are obtained

Where

$\beta 0 = (\alpha_7 - \alpha_5)/(\alpha_8 - \alpha_6)$

$\beta 1 = \alpha_8/(\alpha_8 - \alpha_6)$

$\beta 2 = \rho_1\alpha_5 - \alpha_3 + \alpha_9(1-\rho 1)$

$\beta 3 = \alpha_4 + \alpha_{10}(1-\rho 1)2$

$\beta 4 = \rho_2\alpha_7 - \alpha_9 + \alpha_{11}$

$\beta 5 = \alpha_{12} + \alpha_{10}(1-\rho 1)2$

$\beta 6 = \rho_3\alpha_7 - \alpha_9 + \alpha_{13}$

$\beta 7 = \alpha_{14} - \alpha_{10}(1-\rho 1)2$

$\beta 8 = \beta 2/\beta 3$

$\beta 9 = \alpha 6/\beta 3$

$\beta 10 = \alpha 4/\beta$

$\beta 11 = \alpha 10/\beta 3$

$\beta 12 = (\alpha 1 - \alpha 9)/(\alpha 2 + \alpha 10)$

$\beta 13 = \alpha 2/(\alpha 2 + \alpha 10)$

$\beta 14 = \beta 4/\beta 5$

$\beta 15 = \alpha 8/\beta 5$

$\beta 16 = \alpha 12/\beta 5$

$\beta 17 = \alpha 10/\beta 5$

$\beta 18 = \beta 6/\beta 7$

$\beta 19 = \alpha 8/\beta 7$

$\beta 20 = \alpha 14/\beta 7$

$\beta 21 = \alpha 10/\beta 7$

The cross equation parameter restriction is put and the estimation of the following first four equations is made

$Gs = \beta 0 - (1-\beta 1)Gc * - \beta 1 Gs * + (1-\beta 1)\rho 1\, T + (1-\beta 1)\rho 2\, AG + (1-\beta 1)\rho 3\, AL + \epsilon 1$ [A]

$Gc = -\beta 0 + (1-\beta 1)Gc * - \beta 1 Gs * + \beta 1\rho 1\, T + \beta 1\rho 2\, AG + \rho 3\beta 1 AL + \epsilon 2$ [B]

$T = \beta 8 + \rho 1\beta 9(Gc - Gc*) + \beta_{10}T* + \beta_{11}(1-\rho_1)\, GI - \beta_{11}(1-\rho_1)(1-\rho_2)AG - \beta_{11}(1-\rho_1)(1-\rho 3)AL + \epsilon 3$ [C]

$GI = \beta 12 + \beta 13\, IG * + (1-\rho 1)(1-\beta 13)T + (1-\rho 2)(1-\beta 13)AG + (1-\rho 3)(1-\beta 13)AL + \epsilon 4$ [D]

Equation [A],[B],[C],[D] are estimated simultaneously using cross parameter restrictions.