

## Production Function of the Mining Sector of Iran

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### Abstract

The purpose of this study is to estimate the production function and the structure of production in the mining sector. Several studies have been conducted in the field of estimating production functions of various economic sectors; however, less attention has been paid to the mining sector. By validating variables using augmented Dickey-Fuller and Phillips-Perron tests, this study estimated the production function under different scenarios using co-integration method and time series data for 1976-2006. The unconstrained Cobb-Douglas function in Tinbergen scenario provided better results in accordance with theoretical foundations of economics, statistics and econometrics. Results show that the structure of mining sector of Iran is capital-intensive and work-intensive. Based on findings, the production elasticity of capital and labour are 0.44 and 0.41, respectively. On the other hand, the coefficient of time variable, as an indicator of changes and technological developments in production, is significant representing a positive effect of technical changes on output of the mining sector.

**Keywords:** mine, production function, capital stock, employment, co-integration, productive elasticity

### Introduction

The mining sector is considered as one of the parent industries. In most cases, although it owns a small share in GDP, its effect is considerably on wealth generation. High diversity and abundance of mineral wealth has created a potential for Iranian economy. The mining sector is of great importance in employment and balanced regional economic development. Iran is a rich country in terms of mineral wealth; to use this rich potential for growth and economic development, deserves a considerable thought (Zeytoonnejad, 2005).

Basically, the mining sector refers to that part of the economy which explores, exploits and mineralizes. Based on this definition, measures and activities such as smelting, refining, rolling, etc., relate to the mineral industry and not mining sector. Accordingly, the following form can be defined to justify the activities in the mining sector.

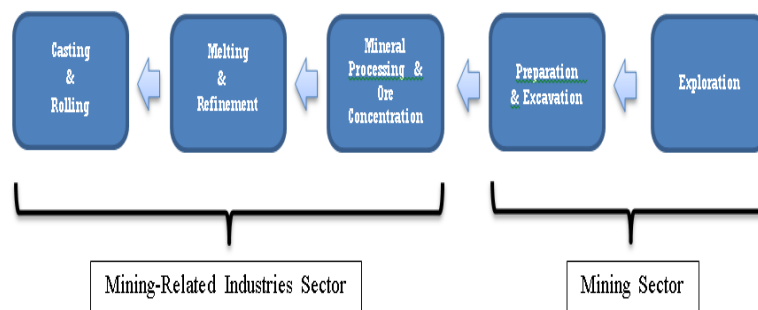


Figure 1: separation of activities of the mining sector and mineral industry

With more than 55 billion tons of confirmed and probable mineral reserves of minerals, Iran is one of the first 12 countries with mineral reserves. However, the mining sector is not in a good place in the economic development of Iran. For many countries, such underground resources are an opportunity for growth and economic development ;however, Iran could not use these opportunities for some reason (Mining and Development, 2005). Lack of studies in regard to production in this sector, perhaps, is one of the missing links in exploitation of opportunities in the mining sector. With a broad perspective, Economics today analyses the problem of production by a technical relationship called as production function.

This article is organized as follows .After the introduction, the research problems are discussed. In the first section, the theoretical basis of the production function is checked. The second section reviews some empirical studies in the field of estimated production function. In the third section, the mining sector is analysed in the Iranian economy. In the fourth and fifth sections, respectively, the model is determined and estimated. Finally, the last section summarizes the conclusions. This article seeks to answer the following questions:

- What is the structure of production in the mining sector?
- What are the effects of technical changes on the mining sector output?

### Theoretical Background

Various ways of combining factors of production to produce goods and services needed by the community is described by production function. This function expresses a technical relationship between inputs and outputs in a simple systematic manner. Production function is a mathematical equation representing the maximum output or product that a firm or a sector of the economy can obtain from any fixed and specified series of production inputs on a certain level of technology. Thus, a production function can be represented as an equation in which the outcomes or the product is considered as the dependent variable and production inputs as independent variables. Accordingly, the production function is as follows:

$$Q = f(L, K, \dots) \quad (1)$$

where, the dependent variable ( $Q$ ) is the output or product and the independent variables are various production inputs, such as labour ( $L$ ), capital ( $K$ ), etc. this function is a neoclassic production function where  $L > 0$  and  $K > 0$ , defined as a continuous function in which at least two derivation is possible. Its partial derivatives are shown as below:

$$\frac{\partial Q}{\partial L} = f_L \quad \frac{\partial Q}{\partial K} = f_K \quad \frac{\partial^2 Q}{\partial L^2} = f_{LL} \quad \frac{\partial^2 Q}{\partial K^2} = f_{KK} \quad (2)$$

Further, assume that:

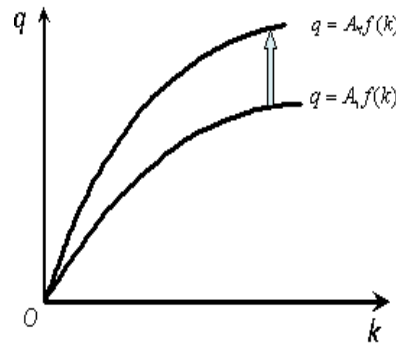
$$f_L > 0 \quad f_K > 0 \quad f_{LL} < 0 \quad f_{KK} < 0 \quad (3)$$

Not, it is ensured that the final production inputs are positive and decreasing.

In any system of production, there are basically two major and distinct concepts in terms of efficiency, one is called technical efficiency and the other allocative efficiency (Libenstein, 1988). In developing equation of production function, it is assumed that engineering and managerial problems of technical efficiency has been previously solved, so that analysis can focus on the problems of allocative efficiency. This is why a correct definition refers production function as a relationship between technical maximum possible yield and the amount of inputs required for producing this product (Shephard, 1970). Despite this, most theoretic and empirical studies define the production function carelessly as a technical relationship between product and inputs and the assumption that such a product is the maximum yield (and minimum inputs) often remains unspoken (Mishra, 2007).

Neoclassical total production function is based on features such as final descent productivity and substitutability of labour and capital together. This function can be obtained with regard to product per capita and capital per worker, as follows.

$$\frac{Q}{L} = A \cdot f\left(\frac{K}{L}\right) \Rightarrow q = A \cdot f(k) \tag{4}$$



**Figure 2: neoclassical total production function**

As shown in Figure 2, the increase in the capital input moves the curve towards higher position (more products), while shifting technology from  $A_1$  to  $A_2$  (assuming  $A_2 > A_1$ ) will move the curve upward. This shift means that the same amount of input achieves more yields. In the economics literature, it is referred to as technical changes.

Total production function is a relationship that is used to describe the technical relations at the macro level. Theoretically, this function is the sum of micro production functions. However, there are piles of works and papers, which were developed from the 1940s onwards, that integrating micro production functions to a macro production function can be quite difficult and problematic (Philip & Fisher, 2003). At least, awareness of these problems seems necessary for every economist who has started practical research. According to Jonathan Temple, every scholar who has decided to specify or estimate the total production function needs to know the circumstances (Temple, 2007). Sylos Labini believes that: it is worth reminding these critiques and criticisms, because a significant and growing number of talented young economists do not know or do not take those criticisms seriously and continues to design and develop various forms of production functions (Sylos Labini, 1995).

This is called aggregation problem in the economic literature. This is partly due to the connections and distinctions between micro and macroeconomics. As long as the production function describes the relationship between yield and inputs of a firm, problems will be more or less organized and problems will be the least, as much as possible. However, when one attempts to identify this in an industry, or sector or the economy, it may be a completely different story. An industry is generally composed of several firms that produce similar products. Of course, each of the firms uses inputs according to the cost, returns to scale, production technology and market conditions.

The total production function of an industry is obtained by relating quantities of the used inputs to total products produced by all firms of that industry. These issues and many other issues give significance to these problems and problems with the use of these functions. It should be noted that the progress toward macroeconomic level (sub-sector, sector and the economy) reveals these

problems (Mishra, 2007). On economics, these problems have been discussed as topics such as investment disputes and Cambridge-Cambridge discussions<sup>1</sup>.

Nevertheless, assuming a total production function still allows our model to be highly predictive and well adapt the total data. As will be discussed below, experience has shown that the total production functions generally provide a good approximation of the real world. In a study published on the new growth theories, Jonathan Temple concludes that although the total production function is the least convincing among all the components and elements of macroeconomics, still it is considered by many economists as the prerequisite knowledge and understanding of national income levels and growth rates (Temple, 1998). Therefore, it should be noted that total production functions could never be accused of theory-less modelling. According to Jonathan Temple (2007), critics of the total production function overemphasize the theory. As a result, there are still compelling reasons for the use of these functions in many areas despite being criticized.

Now a brief introduction to production functions will be presented here. Despite the challenges and criticisms for reasons which were briefly mentioned, production functions have had a strong presence in economic studies and they have been widely used in the economic analysis. Here is a list of names and basic form of these functions.

- Cobb-Douglas production function:

$$Q = A \prod_{i=1}^n X_i^{\beta_i} \quad (5)$$

- Production function with constant elasticity of substitution (CES):

$$Q = A \left[ \sum_{i=1}^n \beta_i X_i^{-\rho} \right]^{-\frac{v}{\rho}} \quad (6)$$

where it is assumed that:  $\rho \geq 1$        $\rho \neq 0$        $v > 0$        $A > 0$

- Translog production function:

$$\ln Q = \alpha_0 + \sum_{i=1}^n \alpha_i \ln x_i + 0.5 \sum_{i=1}^n \sum_{j=1}^n \beta_{ij} \ln x_i \ln x_j \quad ; \beta_{ij} = \beta_{ji} \quad (7)$$

- Transcendental production function:

$$Q = AK^\alpha L^\beta e^{\gamma K + \mu L} \quad (8)$$

- Debertin production function:

$$Q = AK^\alpha L^\beta e^{\gamma K + \mu L + \varepsilon KL} \quad (9)$$

For more information about details and implications of these functions, see Zeytoonnejad (2005).

In this section, theoretical background of production function in general, and a brief introduction of Cobb-Douglas production function, CES, Translog, Transcendental and Debertin specifically were reviewed. Certainly many other mathematical functional forms can explain the production process in sectors or the whole economy by limiting the real world, and adopting certain assumptions. To learn about the history of production functions and their formation, see Mishra (2007). Reviewing the history of production functions, he provided a brief description of

<sup>1</sup>So-called Cambridge-Cambridge discussion was a theoretical debate during 1950s and 1960s between Joan Robinson and allies in Cambridge, England with Paul Samuelson and Robert Solow, in Cambridge, America. In other words, the debate was between Neo-Ricardians (group I) and neoclassic (Group II). Harcourt (1969) presents a full report of what passed between them and their discussion.

features and characteristics of other production functions such as the VES, CMS, GPF, LINEX, multi-output production functions and other production functions.

### Empirical Studies

Production functions are of great importance and frequently used directly in the technical conditions of production, and indirectly in other areas of economics, especially productivity in different economic sectors and levels. Concerning estimation of these functions, several studies have been conducted in different economic levels. These studies are reviewed as follows.

Agheli-Kohneshahri (2006) estimated the production function of mines. He reviewed the consolidated data for mining sector in various provinces during 1996-2002. To estimate the production functions of mines, he logarithmically explained Cobb-Douglas, Translog and transcendental production function. Based on production function estimation by Pooled Least Square (PLS) and Generalized Least Square (GLS), he found that Iranian mines were work-intensive and the factor of production function (The total elasticity of production factors) was greater than 1. This suggests the existence of increasing returns to scale in the mining sector.

Table 1 summarizes the results.

**Table 1: summary of empirical studies regarding estimation of production function**

Author	Country	Time	Level	Data	Inputs	Functions
Agheli-Kohneshahri (2006)	Iran	1996-2002	Mining sector	Consolidated	$K, L$	Cobb-Douglas, transcendental and Translog
Nafar (1996)	Iran	1971-1993	Industries	Consolidated	$K, L$	Translog
Rezapoor & Asefnejad (2005)	Iran	1998-2004	Hospitals	Consolidated	Doctors, nurses, beds, and other staff	Cobb-Douglas
Hadian et al (2007)	Iran	2000-2005	Hospitals	Consolidated	Doctors, nurses, beds, and other staff	Cobb-Douglas
Zeranejad et al (2004)	Iran	1979-2002	Firm	Time series	$K, L$	Cobb-Douglas, Debertin and Translog
Lindenberger (2003)	Germany	1960-1989	Servicing	Time series	$K, L, E$	Cobb-Douglas
Antras (2004)	America	1998-1948	private sector	Time series	$K, L$	Cobb-Douglas
Xiang (2004)	Canada	1997-1961	Macro	Time series	$K, L$	Cobb-Douglas
Khalil (2004)	Jordan	2002	Industrial manufacturing	Sectional	$K, L, M$	Translog
Bonga-Bonga (2005)	South Africa	2002-1972	Macro	Time series	$K, L$	CES
Shankar & Rao (2012)	Singapore	2009-1960	Macro	Time series	$K, L$	CES
Manonmani (2013)	India	2010-1991	Textile industry	Time series	$K, L$	Cobb-Douglas

Estimation results confirmed a different time trend in the technical developments of production. The single time trend (STT) is a very slow upward trend in improvement of production technology during the studied period. However, the multiple time trend (MTT) indicated a very strong reduction in the trend for the year 1971. The results showed diminishing returns to scale in the industry of Iran.

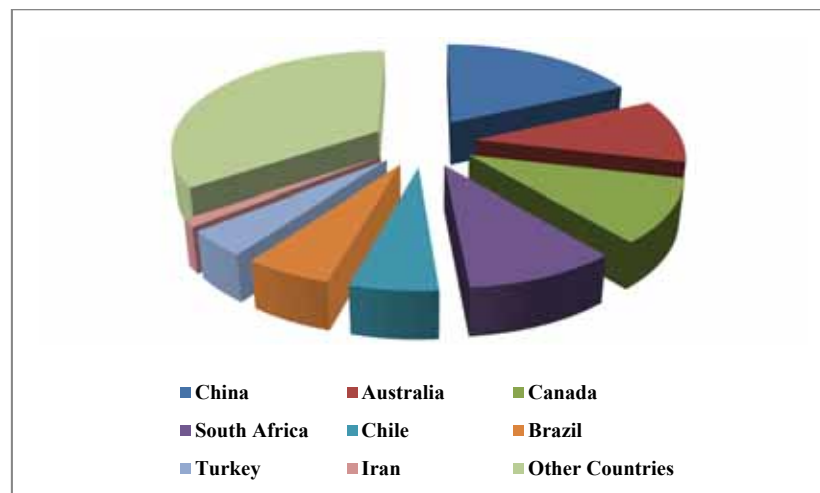
Lindenberger (2003) estimated the production function for service sector of Germany. Forms of functions involved in his research include technological parameters, labour, capital and energy. The results indicate that the time average of production elasticity for capital, labour and energy has been  $\bar{\alpha} = 0.54$ ,  $\bar{\beta} = 0.31$  and  $\bar{\gamma} = 0.21$  during 1960-1978 and  $\bar{\alpha} = 0.53$ ,  $\bar{\beta} = 0.26$  and  $\bar{\gamma} = 0.21$  during 1978-1989.

Shankar and Rao (2012) estimated the long-term growth rates of Singapore. For this purpose, they used a CES production function. The results show that the elasticity of substituting work by capital was 0.6; technical progress in the economy of Singapore was work intensive and long-term economic growth rate was about 1.8%.

### Mining Sector in the Economy

Geological formations of Iran are dominant in the Zagros and Alborz mountains and the formation of central plateau is affected by tectonics. Location of Iran on Alps-Himalayan belt provided its rich resources and a huge variety of minerals. There are 62 types of mature minerals in Iran, which is rare around the world.

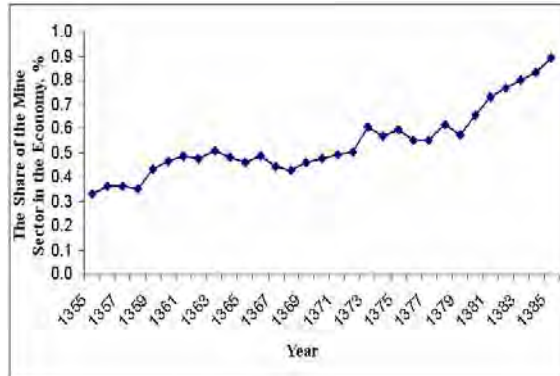
With a vast area, various climates, unique geological and geographical locations, Iran has unique variety of minerals. With more than 55 billion tonnes of confirmed and probable mineral reserves, Iran is one of the top 12 in the world. Based on 2004 statistics and in terms of weight metrics, Iran produces 1.24% of the minerals around the world. Figure 3 shows the production of minerals by the selected countries.



**Figure 3: the fraction of total mineral production for different countries in the world**

However, the mining sector still does not achieve the fraction it deserves in the economy of Iran. Its fraction of GDP is less than 1 percent. The average GDP of Iran during 1976-2006 shows that the average relative fraction of mining sector in GDP has been small (~0.5%), while the fraction of GNP, based on 2004 statistics in South Africa (7.11%), Australia (6.4%), Denmark (5.4%), Mexico (4.3%) and Canada (3%), was much higher than this amount (Zeytoonnejad, 2005).

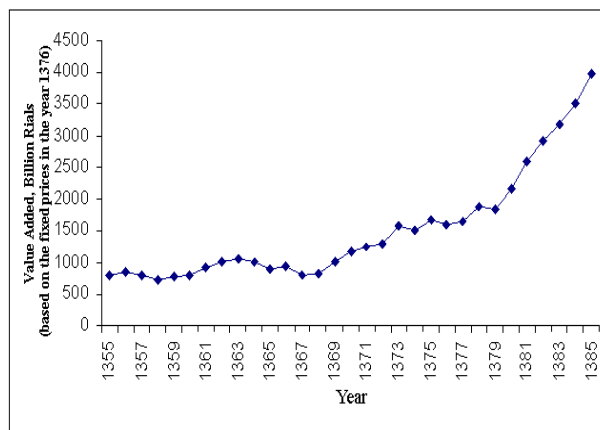
Although the fraction of value-added GDP of mining sector is small in Iranian economy, it has an incremental trend during 1976-2006. To investigate this further, see the diagram below.



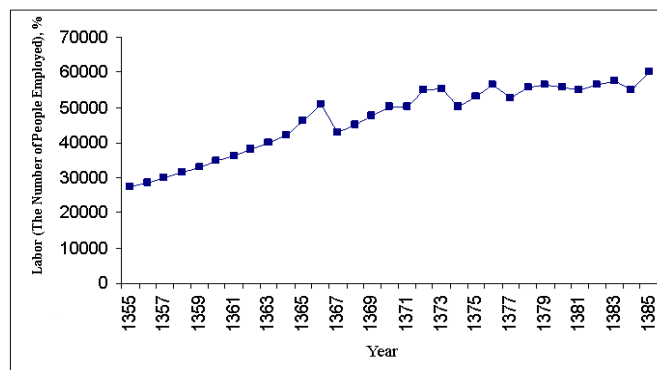
**Figure 4: time trend of value-added GDP for mining sector**

As indicated in the figure above, the contribution of mining sector to GDP has experienced an increasing trend during 1976-2006.

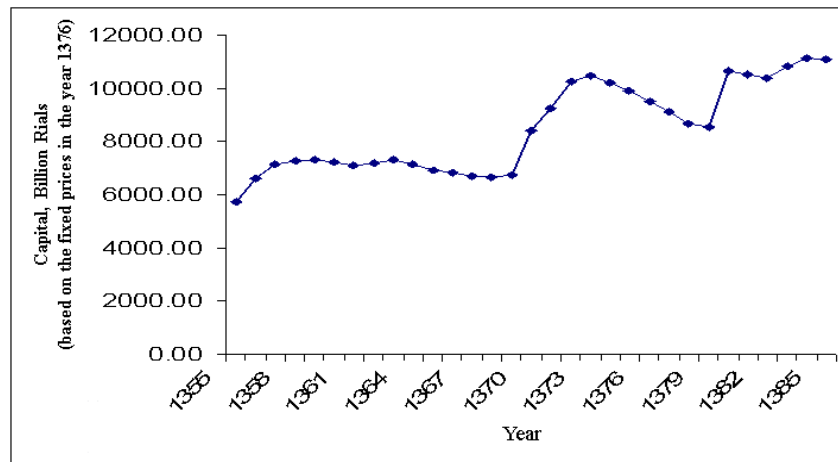
According to the theory of production, the main variables of production are the value-added (as output), labour and capital stock (as input). In this section, we will examine these variables in the period 1976-2006. In this context, time-series diagram of these variables is presented as follows.



**Figure 5: value-added for mining sector during 1976-2006**



**Figure 6: employment in mining sector during 1976-2006**



**Figure 7: time trend of capital stock in the mining sector during 1976-2006**

As the above diagram shows, the value added, employment and capital stock of the mining sector have experienced nearly an increasing trend in a given period. Now after an overview of the variables effective on the production process, the table below reports the annual growth rates of these variables.

**Table 2: average annual growth rate of value-added, labour and capital stock in the mining sector, 1976-2006**

variable	the average annual growth rate
Value-added	5.48%
Labor	2.65%
Capital stock	2.21%

### The Model and Variables

This section first introduces the collected data; then, it estimates production function of the mining sector using Cobb-Douglas, transcendental, Debertin and Translog production functions based on time-series data related to 1976-2006.

Value added is defined as the difference between value of receipts and payments. Value of receipts is indeed the total value of mineral production, saleable mineral waste, construction and major repairs of capital assets by employees and other receipts. In contrast, value of payments is the total value of materials, low-durable tools, consumed fuel, purchased electricity, purchased water and other payments. In this paper, information of national accounts was used to collect data on the value added of the mining sector. National accounts are published annually by the Central Bank. This data was considered as the real value added in fixed prices in 1998.

The capital stock refers to total capital goods, which are measured in a same unit. In mining sector, capital goods are durable machinery, vehicles, equipment, buildings and facilities (disregarding the land value), specific road of the mine, software packages. On the other word, different capital goods are converted to a common unit of measure and are summed together. Accordingly, a measure of physical capital stock is obtained in the mining sector.

In this study, data related to capital stock of mining sector was extracted from estimations of the Central Bank for the period 1975-2002. To estimate the last 5 years (2003-2007), the estimation method of Central Bank was used as follows.



$$K_t = (1 - \delta)K_{t-1} + I_t \quad (10)$$

Where,  $K_t$ ,  $K_{t-1}$  and  $I_t$  are the capital stock in year  $t$ , capital stock in year  $t-1$  and investment in year  $t$  in fixed-prices.  $\delta$  is the depreciation rate which has been considered 4.7% in consistent with Amini (2006).

By definition, labour refers to all persons employed in the mining sector working inside or outside the mine full-time or part-time. Employees are divided into production line employees and administrative, financial and service employees. Productive employees are those involved in the exploration, extraction, mineral processing and deal directly with extraction and production, including Simple and skilled workers, technicians, engineers and transportation staff. Financial, administrative and servicing Personnel include office, administrative, financial and services staff and central office who are not directly involved in the extraction and production.

To collect data on labour, this study used Mines Annual Report, which is published annually by the Statistical Centre of Iran. Although, for this purpose, it was possible to use estimates by the Central Bank, Macroeconomics Office and the Office of Planning and Budget Organization and the various reliable studies, data from the Annual Census of Mines was used because of more credibility of Census data than estimated data.

Unfortunately, there was a gap in the data provided by the Statistical Centre of Iran on the labour employed in the mining sector for years 1977-1984, 1991 and 2005. To obtain the data for these years, interpolation methods were used. In this respect, the data available for the years before and after the gap was used as a benchmark. There are basically two types of interpolation method, exogenous and endogenous. In this study, the exogenous interpolation method was used to address data gaps.

The exogenous approach assumes the average annual employment growth rate as constant and estimates the employment rate regardless of changes in variables effective on employment, such as investment and production. Let  $m$  represent the average annual employment growth rate in two consecutive periods when census has been carried out; employment rate in year  $t$  is obtained as follows:

$$L_t = (1 + m)^t \quad (11)$$

Similarly, few data gaps in the time series of census employment by the Statistics Centre of Iran mines were eliminated.

Therefore, the used variables and data sources are listed as follows.

**Table 3: data resources and definition of the used variables**

Resource	Definition	Variable
Central Bank	Value-added (production)	$Q$
Statistical Center of Iran	Employment	$L$
Central Bank	Capital stock	$K$

As noted earlier, many mathematical functional forms are able to explain production process in economic sectors by limiting the real world and adopting some assumptions. In order to estimate the production function in the mining sector, four different types of production function, i.e. Cobb-Douglas, transcendental, Debertin and Translog as specified below:

**Table 4: summary of explained models for estimation**

Production function	Explained model
unconstrained Cobb-Douglas (C-D)	$\ln Q = \alpha + \beta \ln K + \theta \ln L$
Unconstrained Tinbergen C-D	$\ln Q = \alpha + \beta \ln K + \theta \ln L + \gamma T$
Constrained arbitrary C-D	$\ln\left(\frac{Q}{L}\right) = \ln A + \alpha \cdot \ln\left(\frac{K}{L}\right)$
Constrained Tinbergen arbitrary C-D	$\ln\left(\frac{Q}{L}\right) = \ln A + \alpha \cdot \ln\left(\frac{K}{L}\right) + \gamma T$
Transcendental	$\ln Q = \alpha + \beta_1 L + \beta_2 \ln L + \theta_1 K + \theta_2 \ln K$
Debertin	$\ln Q = \alpha + \beta_1 \ln L + \theta_1 \ln K + \beta_2 L + \theta_2 K + \delta KL$
Translog	$\ln Q = \alpha + \beta_1 \ln L + \beta_2 (\ln L)^2 + \theta_1 \ln K + \theta_2 (\ln K)^2 + \delta (\ln L)(\ln K)$

In the explained production functions, coefficients of the variable trend are considered as a cause of technical changes and a positive sign of the coefficients indicates the technical progress. Cobb-Douglas model can be estimated under several scenarios. This function is estimated in constrained and unconstrained as well as simple and Tinbergen forms.

### Experimental Results

Among above models, the results of some models were not consistent with theories of economics; therefore, those results were avoided to report. In most of these models, large number of explanatory variables reduces the degrees of freedom and causes non-compliance with the principle of scarcity (saving) followed by some problems; on the other hand, repeating variables in different ways causes co-linearity problems for the model and influences the standard deviation and statistical significance of the variables. It is noteworthy that the variable virtual war was also inserted in the models and it was found that it was not significant in any of the above models.

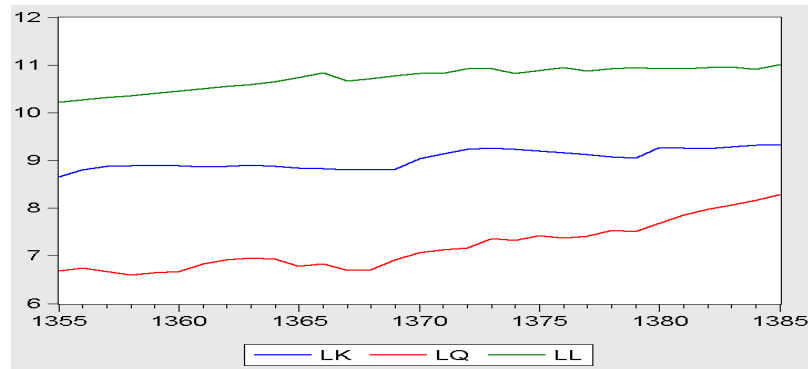
In this study, the unconstrained Tinbergen Cobb-Douglas model provided the best results in compliance with theories of the economics and in consistent with statistic and economic measures. Before estimation, the first step is to test the reliability of variables. Then, the production function of mining sector is estimated. Next, the assumptions and requirements are evaluated by proper tests.

#### Reliability

The economic analysis assumes that there is a long-term balanced relationship between variables considered in an economic theory. In practical econometric analysis to estimate long-run relationships between variables, the mean and variance are constant over time and independent of the time factor. Therefore, a behavioural consistency is implicitly assumed for them. However, research has found that the behavioural consistency of the time series variables is not fulfilled in most cases.

Therefore, the classic t and F tests of estimation methods, in which behavioral consistency or the so-called reliability of variables is not fulfilled, are not validated and the results will be misleading. This problem is referred to as fake regression. Consequently, variables need to be tested for reliability in order to ensure the results.

A time series variable is reliable when the mean, variance and autocorrelation coefficients remain constant over time. In other words, if starting time of a series changes and mean and variance-covariance remain unchanged, the series will be reliable. A graph of logarithmic time-series variables used in this study is presented below.



**Figure 8: the logarithmic values of the time series variables used in the models**

Figure (8) implicitly suggests a time trend and that above variables follow the time factor. In other words, this graph implicitly indicates unit roots in the variables above. To test this, we used the following tests, in order to be explicit about the reliability of the above variable.

Unit root tests such as Augmented Dickey-Fuller (ADF) and Philips-Perron (PP) were used for this. The results show that variables are all  $I(1)$ . Summary of results is presented in Table (5).

**Table 5: summary of results from reliability tests conducted for logarithm of variables**

Variable	ADF level	PP level	First order difference of ADF	First order difference of PP	Result
LL	-2.157	-2.071	-6.257	-10.904	□□□□
LK	-2.751	-2.472	-4.124	-4.114	□□□□
LQ	-1.120	-1.120	-4.979	-4.984	□□□□

While testing levels, the critical values at 1%, 5% and 10% were -4.297, -3.568 and -3.218, respectively; for the first order difference, the critical values at 1 %, 5% and 10% were -4.310, -3.574 and -3.222, respectively.

Now, the production function is estimated using the ordinary least squares estimator. In general, these estimators as Gauss-Markov theorem are the best linear unbiased estimators (BLUE). However, this method can be used to estimate the coefficients when the model satisfies the following assumptions:

- Lack of bias ( $E(u_i) = 0$ )
- Lack of variance anisotropy ( $E(u_i^2) = \sigma^2 I$ )
- Lack of autocorrelation of disturbing elements ( $E(u_i, u_j) = 0$  in  $i \neq j$ )
- Lack of correlation between disturbing elements and explanatory variables ( $E(X_i u_i) = 0$ )
- Normal distribution of disturbing elements with variance  $\sigma_u^2$  and the expected value of zero ( $u \sim N(0, \sigma_u^2)$ )

In estimating the production function of mining sector, the best model based on theoretic econometrics is the unconstrained Tinbergen Cobb-Douglas production function. Considering the conditions above, we have attempted to estimate this function. Table 6 summarizes the results of the estimation of the production function:

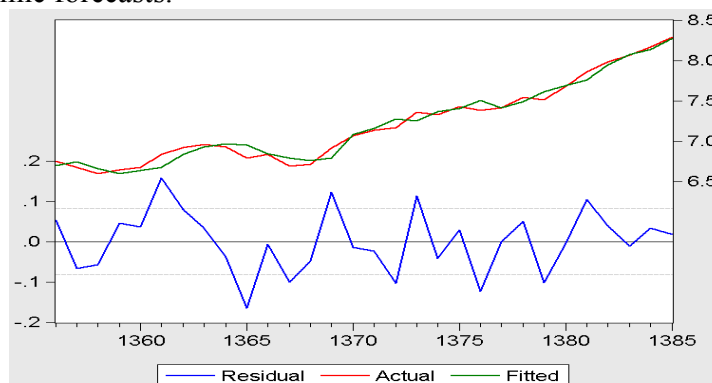
<sup>2</sup> However, the same tests of the reliability were conducted on logarithmic values of capital and value added per capita; the results of both tests showed the reliability was  $I(1)$  for both variables.

**Table 6: summary of results from estimation of unconstrained production function for mining sector**

Parameters, measures, statistics	Values
Elasticity of production to capital) $\alpha$ (	0.44
<i>t-Statistic of <math>\alpha</math></i>	1.88
Elasticity of production to labor ( $\beta$ )	0.41
<i>t-Statistic of <math>\beta</math></i>	1.47
Factor of technical change	0.08
<i>t-Statistic related to factor of technical change</i>	1.67
The coefficient of AR (1)	0.9
<i>t-Statistic related to coefficient of AR (1)</i>	10.58
$R^2$	98%
$\bar{R}^2$	97%
<i>F-Statistic</i>	255
Durbin-Watson	1.93
Number of observations (n) after adjustment	30

The results from this estimation show that elasticity of production to capital and labour was 0.44 and 0.41, respectively. The factor of technical changes was positive and significant in the model, which means the positive impact of technological change on output of the mining sector. To solve the problem of autocorrelation in the model, the component AR(1) was added to the model. In these circumstances, the Durbin-Watson (DW) approached the number 2 suggesting no autocorrelation between disturbing elements. The coefficient of determination and the adjusted coefficient of determination were 98% and 97%, respectively, indicating the good explainability of the model. In other words, the coefficient  $R^2=985$  indicates that 98% of variation in the dependent variable (value-added) can be explained by the explanatory variables. Significance of coefficients (t) was also good, so that coefficients of capital (95% confidence) and coefficients of labour and time trend (90% confidence) were significant. Numerical value of f-statistic indicates the significance of the model as a whole.

To see the actual graph of value-added and the fitted line (estimated regression line) as well as the plot of error terms, refer to. Partial compliance and proper fitness of the regression line on the graph of value-added indicate the suitability of the specified model and ability of the estimated relationship in economic forecasts.

**Figure 9: fitness of the estimated unconstrained production function on the graph of value-added with the graph of disturbing elements**

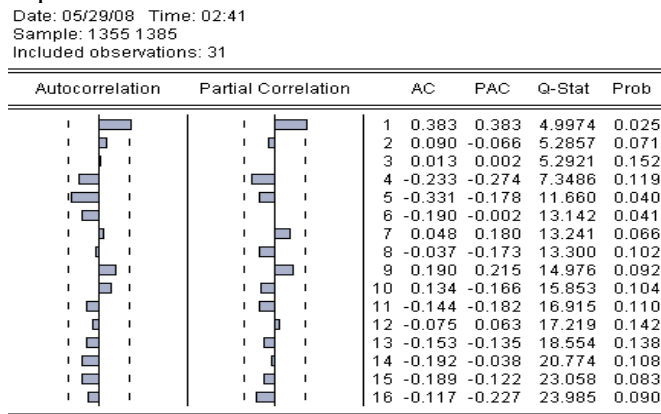
Thus, indices of the estimated model are acceptable according to statistical, theoretical and econometric criteria; therefore, the estimated model can be accepted as a schema. Now, the schema is evaluated for classical assumptions such as variance anisotropy, normality, etc. The degree of reliability and validity of the model will be determined accordingly.

**Anisotropy Analysis of Variance**

One of the classic assumptions is equal variances of the disturbing elements in different periods. In other words,  $E(u_i^2) = \sigma^2 I$  where  $i = 1, 2, \dots, n$ . Violation of this assumption creates a problem called variance anisotropy. By testing variance anisotropy of regression through Breusch-Pagan-Godfrey method, it was determined that the assumption of variance anisotropy could be rejected. Therefore, there is no problem of variance anisotropy in the estimated model.

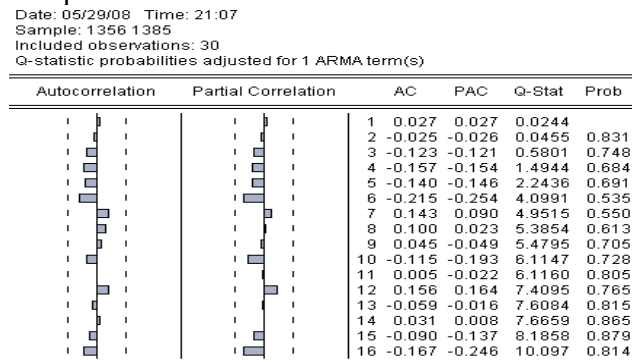
**Autocorrelation test**

To investigate the problem of autocorrelation, Breusch-Godfrey test (LM test) was used. The results of this test for the primary unconstrained model (before inserting AR(1)), implies the problem of autocorrelation in the estimated model. The diagram of autocorrelation and partial correlation of the disturbing elements is presented as follows.



**Figure 10: the autocorrelation and partial correlation of disturbing elements in the primary unconstrained model**

Autocorrelation problem is clearly visible in the graphs above. In order to solve this problem, an AR(1) was added to the model, and thus the problem was resolved. Results of Breusch-Godfrey test, after inserting AR(1) into the model, suggest that the problem was resolved. Accordingly, the diagram of autocorrelation is presented as follows.

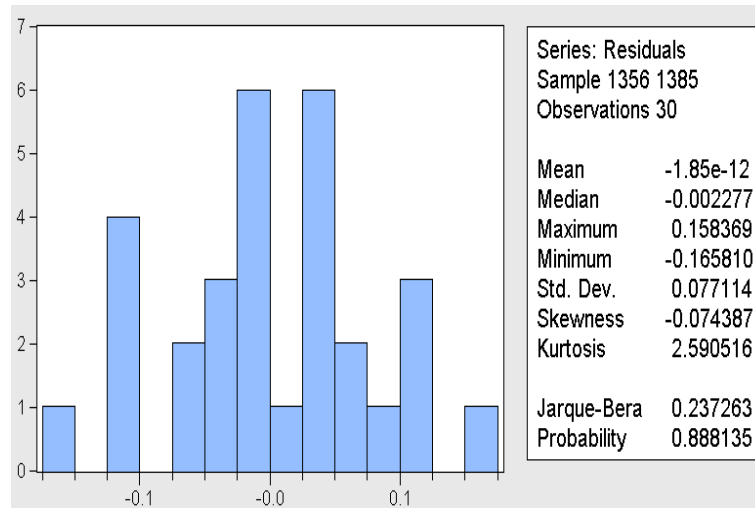


**Figure 11: autocorrelation and partial correlation of disturbing elements in the final unconstrained model**

Therefore, adding the component AR(1) to the model resolved the autocorrelation problem. We will continue to investigate other assumptions of the classical regression model.

#### ***Normality of the disturbing elements***

The results showed that disrupting elements were distributed normally. To explore this, see the diagram (12).



**Figure 12: the results obtained from the statistical distribution of disturbing elements in the unconstrained model**

Thus, Figure 12 shows that the assumption of normal distribution for the random process of disturbing element is true; in other words,  $u \sim N(0, \sigma^2)$ . This in turn suggests  $E(u_i) = 0$  which indicates that another classical assumption is also true. Calculation of  $E(u_i)$  shows that its value is  $-1/85186E-12$  which is almost equal to zero.

#### ***Independence of disrupting elements from the explanatory variables***

Results from calculating the coefficient of correlation between disturbing elements and explanatory variables of capital stock and labour were 0.054 and -0.098, respectively, which suggest the independence of disturbing elements from both explanatory variables in the estimated model. Thus, the assumption of independence of disrupting elements from the explanatory variable is true. Therefore, it was determined that all the classical assumptions are true in the estimated model and there is no violation of the classical assumptions.

#### ***Co-linearity***

Co-linearity originally means a linear relationship and strong correlation between the explanatory variables of the regression model. Even if the co-linearity is too severe, OLS estimators maintain the BLUE feature. Some researchers fear that the explanatory variables are co-linear, but co-linearity does not violate any of the assumptions of regression. Unbiased and consistent estimators and their errors are correctly estimated. The only effect of co-linearity is that estimations of the coefficients are obtained with large standard errors. As this effect is observed for the independent variables with small variances, this is also true for small number of observations. Empirical rules based on which the presence or absence of co-linearity can be determined are as follows:

- The high  $R^2$  and the low number of significant t ratios
- High paired correlation between the explanatory variables

Review of the empirical regularities found no severe co-linearity in the model. Now as the final stage of co-integration, it is necessary to calculate the time series of disturbing elements and test the reliability of this series. Series of the disturbing elements was obtained using outputs of Eviw6 software. As the last step of co-integration, the time series of disturbing elements is examined for reliability. Only if the time series is reliable, the results of ordinary least squares estimation are validated. Using both ADF and PP tests, reliability of the series was investigated. Summary of the results is listed in Table 7.

**Table 7: summary of results from reliability tests for disturbing elements of the unconstrained model**

Variable	ADF level	PP level	Result
Disturbing elements	-5.096	-5.093	□□□□

The results from both methods suggest that the series is reliable. Thus, the estimated model is acceptable as a schema.

### Conclusion

Despite the low contribution of mining sector to GDP, its effect is considerable on the secondary production of wealth. High diversity and abundance of mineral wealth in the Iranian economy has created a considerable potential. The mining sector, as the main source of primary materials needed for important industries and besides employment and balanced regional economic development, is of great importance. However, the economic exploitation of these exhaustible resources is possible by optimal composition of the primary factors of production, namely labour, machinery, intermediate goods and energy.

By reviewing theoretical background, empirical literature and status of the mining sector in the Iranian economy, this study estimated the production function of mining sector and revealed the mediatory structure of production in mines of Iran. In other words, the structure of Iranian mining sector is work-intensive and capital-intensive. This is because of the fact that elasticity of production to capital and labour has been 0.44 and 0.41, respectively, which are not considerably different. On the other hand, the production is in economic zone to both inputs. The factor of function has been 0.85 for this sector, indicating the decreasing return to scale. Moreover, the coefficient of trend, as the index of technical changes in production, was significant, suggesting the positive effect of technical changes on output of the mining sector.

Therefore, to promote level of technology, to create stability in order to provide a bed for investment, to develop infrastructures, to make policies to increase incentives, to increase R & D, to revise scales of production<sup>3</sup> in order to use economies of scale and spread of information banks of Geology and exploration can be helpful in the efficiency of this sector.

### Resources

- Agheli-Kohneshahri, L. (2006). Estimation of Production Function in Iranian Mines. *Economic Studies*, 6(1), pp. 33-50.
- Amini, A. (2000). Estimation of time series of employment in economy of Iran during 1966-1996. *Planning and budget*, 51, 35-67.
- Amini, A. & Neshat, H. M. (2005). Estimation of Time Series of Capital Stock in Economy of Iran during 1959-2002. *Planning and Budget*, 90, 53-86.

<sup>3</sup>According to economic theory, the optimal scale of production is a level of production in which the average cost (AC) of production is minimal.

- Amini, A., Safaripour, M. & Nahavandi, M. (1998). Estimation of time series of employment and capital stock in economic sectors of Iran. *planning and budget*, 31-32, 69-97.
- Anon., n.d. annual report of mines. [Online] Available at: <http://amar.sci.org.ir> [Accessed 12 January 2009].
- Anon., n.d. database of economic time series. [Online] Available at: <http://tsd.cbi.ir>[Accessed 12 January 2009].
- Hadian, M. (2008). Production function of Hospitals supervised by urumie university of medical sciences. *health management*, 29, 7-14.
- Lotalipour, M. (1991). production functions of mining sector in economy of Iran, Tehran: s.n.
- Nafar, N. (1996). Estimation and Analysis of Production Function in Industries of Iran. *Economic Studies of Modares*, Issue 1, pp. 63-86.
- Rezapour & Asefzadeh (2006). estimation of production function in hospitals supervised by qazvin university of medical sciences. *journal of qazvin university of medical sciences*, 10(40), 86-90.
- Zeranejad, M. & Loraki Bakhtiarinejad, M. (2002). estimation of production function for pipe manufacturing company of Ahvaz. *economic studies of Iran*, 19, 117-141.
- Zeytoonnejad, A. (2005). The role of mines on national economy, Tehran: s.n.
- Zeytoonnejad, A. (2009). Estmation of production function and measurement of total productivity growth of factors in mining sector of Iran, firouzkouh: s.n.
- Zeytoonnejad, A. & Mahmoudzade, M. (2013). Measurement and analysis of resources of economic growth in minig sector of Iran. *macroeconomics*, 13, 121-142.