European Online Journal of Natural and Social Sciences 2014; www.european-science.com
Vol.3, No.3 Special Issue on Environmental, Agricultural, and Energy Science
ISSN 1805-3602

Numerical Modelling of Hydraulic Flow in Dam Stepped Spillway and Study of Cavitation Phenomenon

Seyed Payman Karami[1*], Amir Khosrojerdi[2], Mahmood Shafai Bajestan[3]
[1]Hydraulic Structures, Azad University, International Branch, Kish; [2]Research and Science Institute, Tehran; [3]Water Engineering Sciences, Shahid Chamran University, Ahvaz
*E-mail: paymank9@gmail.com

Abstract

Cavitation is a phenomenon leading to damaging surfaces and usually occurs at steep and fast overflows. This can be prevented via decreasing pressure upon the floor of overflows or controlling the flow rate. Furthermore, controlling this phenomenon comes about through increasing air concentrations in the overflow. One of the methods of increasing air concentrations and decreasing the flow rate of the spill is to construct overflows in stepped forms. Dam spillway releases are from among shoot overflows which would confront cavitation phenomenon. This research tries to control this phenomenon via constructing stepped spillways instead of installing aeration systems. Because laboratory models and field researches are costly and time-consuming, using numerical modelling via CFD method for designing and simulating hydraulic flows are suggested. In this research, stepped fast flowing dam releases were designed instead of spillway fast flows, and then simulating flows on spillways, the cavitation phenomenon was studied and evaluated. Results from these simulations show that constructing stepped forms upon dam spillway releases are effective in controlling cavitation phenomenon, so that via reducing flow rate and increasing cavitation index, prevention of this phenomenon becomes possible.

Keywords: Stepped spillway, Cavitation, Flow Modeling

Introduction

Most of engineering fields, which concern fluids and their characteristics, focus on dynamic behavior of fluids. One of these fields are called Hydraulics which are mainly related to fluid weather. From among these, different natural and unnatural factors can play an important role in determining characteristics of flows.

Experimental and laboratory studies about such flows have long been of utmost importance. In laboratory researches, characteristics of flows are studied based on the special model and results are presented as physical relations via analyzing dimensions of it. Using laboratory models is costly and time-consuming.

The other method for studying flows is performed via using computational fluid dynamics which is based on numerical methods. Moreover, spillways of most medium and large dams of the world concern flow rates of more than 10 m/s to 45 m/s. When flow rate gets close to 10 to 15 m/s, cavitation phenomenon becomes more and more serious. The existence of small bumps at surface of concrete may cause cavitation and damaging concrete at downstream levels. Therefore, for preventing such phenomenon, well-constructed details for concrete floors should be considered. If flow rates exceed 20 to 30 m/s, the probability of occurring cavitation increases, therefore concrete would require more protective operations. The most effective method of decreasing the effects of cavitation in damaging concrete is via using ventilation. Ventilation is of two kinds: one method is using places to ventilate the shoot spillways; another one is using stepped spillways which lead into natural ventilation.

Review of Literature

Kisidi et al (1965) were from among the first who used Laplace Equation for investigating spillways flows. They used *Finite Difference Method* in computing page for determining variables, and he used *Methods for Complex Arguments* for converting physical page into computing page.

Ikgava and Vashizo (1973) were from among the first who used *Finite Element Method* to solve Laplace Equations at spillways.

Diresh et al. (19770 used finite element method with flexible grids (non-rigid) for solving potential flow equations at spillways. Lee and Chen (1989) performed their research upon three heads of 0.5 Hd, Hd and 1.33 Hd at WES spillway using Limited Analytical Method. Bergisser and Ranchman in 1999 used finite element method to solve Navier-Stokes equations, considering Constant viscosity at about 0.001 $\mathrm{m}^2/_{s}$. Assi (2001) solved potential flow equation at spillways using limited difference method, considering another forms for Neumann boundary conditions. Savage and Johnson (20010 used Reynolds Average Equations for analyzing two dimensional flows using FLOW-3D Software. Sorensen (1985) chose stepped spillways as the best choice via analyzing construction of the physical model, Cup Launch, Stilling Basin and Shoot in stepped forms.

Rajaratnam (1990) presented the following equation for introducing Coefficient of Friction in stepped spillways considering a non-spilling flowing regime:

$$C_f = \frac{2{y_n}^3 g \sin\alpha}{q^2}$$

In which α is slope of the spillway, g acceleration of gravity, q overflow discharge per width and y_n is normal depth.

Christo Dolo (1993) investigated the effects of steps in energy dissipation at stepped spillways. Furthermore, he showed the results of his experiments and those of Sorensen (1985) in the following figure.

.

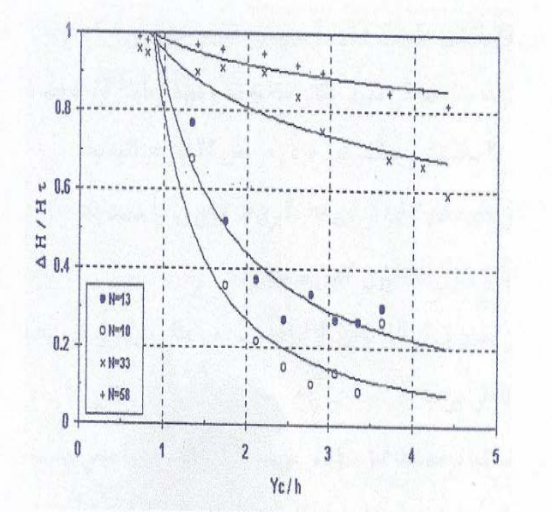

Figure 1. The effects of steps in energy dissipation at stepped spillways

In this figure, the effects of the number of steps upon energy dissipation is clear, and per each amount of $\frac{y_c}{h}$, the amount of energy increases by adding to the number of steps. Moreover, the effects of the number of steps becomes very clear at high discharges.

Chanson (1994) via analyzing the results obtained by other researchers showed that for non-spilling flow regimes come true, flow rate should exceed a certain amount. Rice and Cadovi (1996) studied a physical sample, at a scale of 1.20 to estimate flow transition at spillways. Chen et al (2002) used fluid volume models for numerical simulation of turbulent flows at stepped spillways. Bows and Hedger (2003) performed experiments about flows passing from stepped spillways and reported results about velocity distribution and air concentration in their reports. Tabtra et al (2005) used adina software to analyze stepped spillways via FEM, and used standard k-ε model to determine turbulent flow. Sanchez et al. (2007) analyzed pressure field at stepped spillway of roller compacted concrete dams. Chanson (2008) expressed his ideas about the effects of scale in samples of stepped spillways and said that the complexity of two-state flows at stepped spillways increase the scale effects more and more. Zhang Dong et al (2009) compared different flows at numerical models and laboratory samples to conclude that k-ε model is effective in simulating flows at stepped spillways. Karvaly and Martinez (2009) analyzed Hydraulic jump in stepped spillways in analytical, physical and numerical. Salmasi et al (2003) investigated the effects of the number of steps on energy loss in laboratory samples of stepped spillways and concluded that the relative rate of energy dissipation decreases by increasing flow rates. Samani et al (2004) used laboratory samples to investigate principles upon Hydraulic flows and designing stepped spillways and obtain an equation for calculating energy loss. Fathi et al (2009) used laboratory methods to study the effects of the slope of the downstream head upon the place where natural air is absorbed. Ahmad Yar et al. (1993) performed their research on physical samples of steep stepped spillways. Choosing four heights for steps such as 40, 30, 24 and 20 mm, and changing four angles of the bottom of steps 25, 20, 15 and 10 degrees, they investigated 16 physical samples of stepped spillways. Mousavi et al. (2008) simulated Hydraulics of flows at stepped spillways in both spilling and non-spilling steps using physical models and ANSYS software. They concluded that ANSYS is able to investigate different Hydraulic dimensions of stepped spillways. Ashourian (2013) compared results from numerical models with those obtained from physical models using FLOW-3D software to show that energy loss increases when the height of steps increases. Hossein Nouri (2013) modeled flows on shoot spillways and compared it with results from physical model to investigate cavitation phenomenon and determine places which require ventilation. Sar Afraz, Attari and Ma'aroufi (2011) used numerical FLOW-3D model to determine the starting ventilation point at a gentle slope of a stepped spillway. Pirestani et al (2010) numerically modelled flows at stepped spillways and compared it with physical model via using FLUENT software.

Material and Methods

Calculating dimensions and the number of steps for numerical modelling

The model for spillway dam releases and results for Shoot spillways are available at Institute of Water Researches. Spillway dam releases are determined based on stepped spillway design relations compared to the design of the dimensions of optimized steps. From another point of view, because these researches aim at comparing shoot spillways with stepped ones using results from shoot spillways, therefore, the dimensions of the spillways will remain constant, and just shoot changes into step. So, we have:

$$S = 36.4\% \quad , \quad \theta = 20^\circ$$

From another point of view, the width of the main spillway is 30 m and the width of the model is 0.9 m, so we have:

$$M = \frac{30}{0.9} = 33.33$$

This means that the main spillway is 33.33 times larger than the model. The geometric similarity relations for flow rate, discharge and length are as follows:

$\frac{Q_P}{Q_M} = M^{2.5}$ $\frac{V_P}{V_M} = M^{0.5}$, $\frac{L_P}{L_M} = M$

PMF discharge for dam release is 2290 m^3/s, discharge of 10000 years for 1545 m^3/s. First for this discharge the maximum amount of flood which is about 2290, we would have:

$$Q_M = \frac{Q_P}{M^{2.5}} = \frac{2290}{33.33^{2.5}} = \frac{2290}{6413.4} = 0.357 \quad \text{cms}$$

$$q = \frac{Q_M}{b_M} = \frac{0.357}{0.9} = 0.397 \quad m^2/s$$

For designing spillways, we do as follows. We use the following equation to obtain critical spillway depth.

$$d_c = \sqrt[3]{q^2/g} = \sqrt[3]{0.397^2/9.81} = 0.252 \quad \text{m}$$

Based on researches, the optimized conditions for the height of steps are as follows: the most optimized heights are 30, 60, 90 or 120 cm. we consider 90 cm and calculate the height of model:

$$L_M = \frac{L_p}{M} = \frac{0.9}{33.33} = 0.027 \text{ m} = 2.7 \text{ cm}$$

So, our first assumption is that the height of steps are 2.7 cm.

At next stage, we should determine the height of steps for occurring non-spilling regime so that $0.25 \leq h/d_c \leq (h/d_c)_s$ in which $(h/d_c)_s = \frac{7}{6}(\tan \alpha)^{1/6}$ for $5.7^\circ < \alpha \leq 55^\circ$ would be:

$$(h/d_c)_s = \frac{7}{6}(\tan \alpha)^{1/6} = \frac{7}{6}(\tan 20)^{1/6} = \frac{7}{6}(0.364)^{1/6} = 0.99$$

Therefore, $0.25 \leq h/d_c \leq 0.99$ should be considered.

But, while $h/d_c = 0.027/0.252 = 0.11 < 0.25$ is not true about the above conditions, then we have to add the height of steps or decrease discharge designs. While the height of 90 cm is reasonable for steps and more height is not economical from an engineering point of view, therefore, considering constant height and doubling it for different discharges, we would test the above condition.

Table 1. Height of steps in different designs

800	800	900	900	1545	1545	2290	2290	Qp
0.125	0.125	0.140	0.140	0.241	0.241	0.357	0.357	Qm
0.139	0.139	0.156	0.156	0.268	0.268	0.397	0.397	Q
0.125	0.125	0.135	0.135	0.194	0.194	0.252	0.252	Dc
0.036	0.027	0.027	0.036	0.054	0.027	0.054	0.027	H
0.288	0.216	0.200	0.266	0.278	0.139	0.214	0.107	h/dc

Based on the above table for discharge of 2290 m^3/s, the height of steps should be more than 180 cm and if the height of steps is more than 180 cm, then the discharge of 1545 m^3/s would meet the primary conditions. But as was mentioned previously, the optimized height of steps is maximally 120 cm. Therefore, we cannot design stepped PMF discharge designs for the above spillways.

From another point of view, the discharge in unit of width for stepped spillway should not be more than 30 m/s. this means that for width of spillway (B), we would have:

$$q = \frac{Q}{B} \leq 30 \qquad {m^2}/{s}$$

If we consider q as 30 m/s in prototype, and while the width of spillway is 30 m, then optimized discharge that satisfy conditions for discharge in unit of width are as follows:

$$Q_P = q_P * b_P = 30 * 30 = 900 \qquad {m^3}/{s}$$

$$Q_M = \frac{Q_P}{M^{2.5}} = \frac{900}{33.33^{2.5}} = \frac{900}{6413.4} = 0.14 \quad \text{cms}$$

$$q_m = \frac{0.14}{0.9} = 0.156 \qquad {m^2}/{s}$$

$$d_c = \sqrt[3]{q^2/g} = \sqrt[3]{0.156^2/9.81} = 0.135 \quad \text{m}$$

Modelling in FLOW-3D software

For modelling in FLOW-3D software similar to laboratory samples, we consider the following assumptions which gives a sufficiently exact simulation of different discharges.

Table 2. The dimensions for experimental model in this research

Profile sphere mesh		
Mesh block	*Meshing at x direction*	*The length of cell*
Mesh block 1	*0-2.80*	*360,000*
Mesh block 2	*2.80-5*	*720,000*
Mesh block 3	*5-8.23*	*1,080,000*
Whole cells in meshing site		*2,160,000*

- *RNG model*
- *Fluid: water at 20^o C*
- *Gravity -9.81 at Z axis*
- *Primary conditions: Reservoir is always full of water at the height of upstream*
- *Border conditions: according to the defining three mesh blocks, the border conditions for each are as follows:*

Table 3. Numerical method: GMRES

	Mesh block		
	Mesh block 1	*Mesh block 2*	*Mesh block 3*
Xmin	*Volume Flow Rate*	*Symmetry*	*Symmetry*
Xmax	*Symmetry*	*Symmetry*	*Outlet*
Ymin	*Wall*	*Wall*	*Wall*
Ymax	*Wall*	*Wall*	*Wall*
Zmin	*Wall*	*Wall*	*Wall*
Zmax	*Symmetry*	*Symmetry*	*Symmetry*

Results

While physical model for shoot spillway is designed in Institute of Water Researches, all features of the spillway are considered similar and only shoot is changed into step, therefore, considering different discharges and different dimensions for steps, this spillway is investigated and the optimized model is obtained. Features for these models are mentioned in the following tables:

Table 4. Some features of these models

Scale 1:33.33	*Q(m3/s)*	*q(m2/s)*	*Yc(m)*	***The Height of the Head for still overflow (m)***
Prototype	*2290*	*76.3*	*8.406*	*180.53*
Physical model	*0.357*	*0.397*	*0.252*	*5.42*
Prototype	*1545*	*51.5*	*6.466*	*180.53*
Physical model	*0.241*	*0.268*	*0.149*	*5.42*
Prototype	*900*	*30*	*4.510*	*180.53*
Physical model	*0.14*	*0.156*	*0.135*	*5.42*
Prototype	*800*	*26.7*	*4.170*	*180.53*
Physical model	*0.125*	*0.139*	*0.125*	*5.42*

Results obtained from discharges on spillways with different heights of the steps are shown in the following graphs:

Comparing flowing depth in different discharges, with 90 cm height of the step

Comparing flow rate in different discharges with 90 cm height of steps

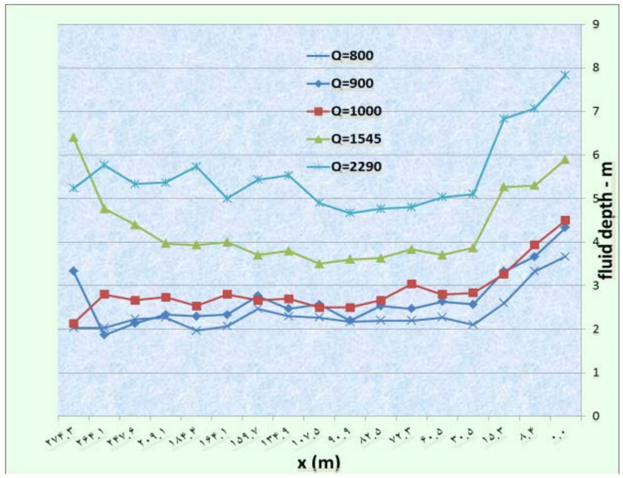

Figure 2. Comparing flowing depth in different discharges at 120 cm height of the step

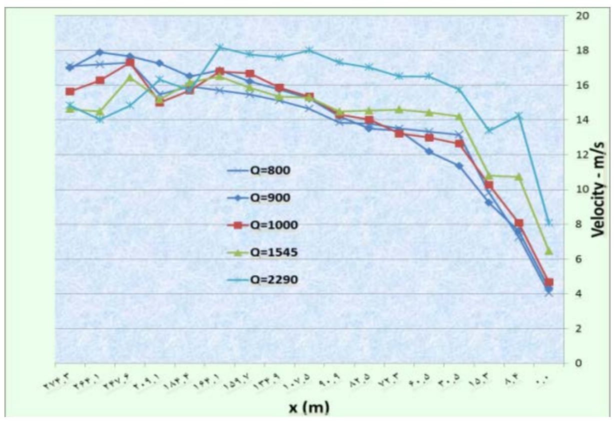

Figure 3. Comparing flow rate in different discharges at 120 cm height of the step

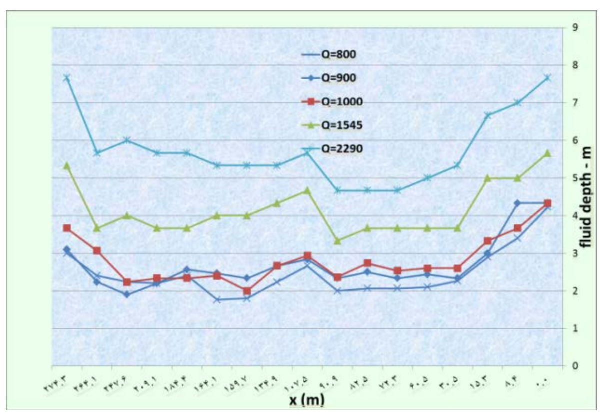

Figure 4. Comparing flowing depth in different discharges at 180 cm height of the step

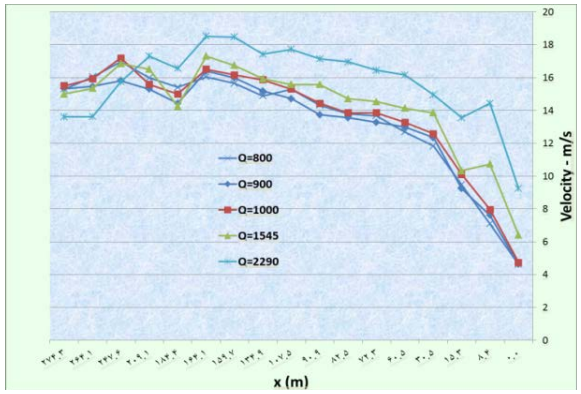

Figure 5. Comparing flow rate in different discharges at 180 cm height of the step

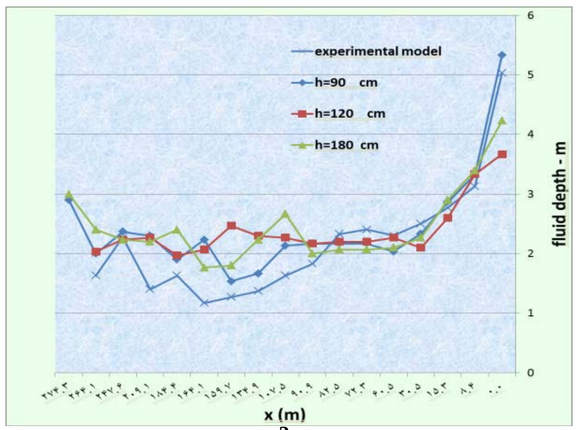

Figure 6. Comparing flowing depth at 800 m^3/s discharge and different heights of the steps

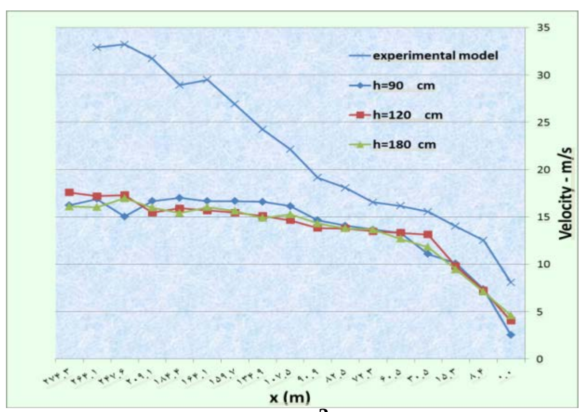

Figure 7. Comparing flow rate in 800 m^3/s and different heights of the steps

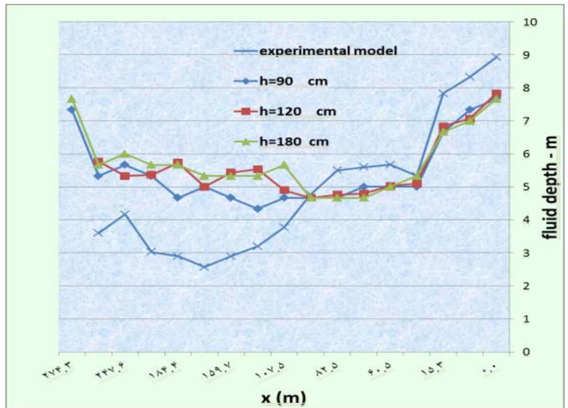

Figure 8. Comparing flowing depth at 2290 m^3/s discharge with different heights of steps

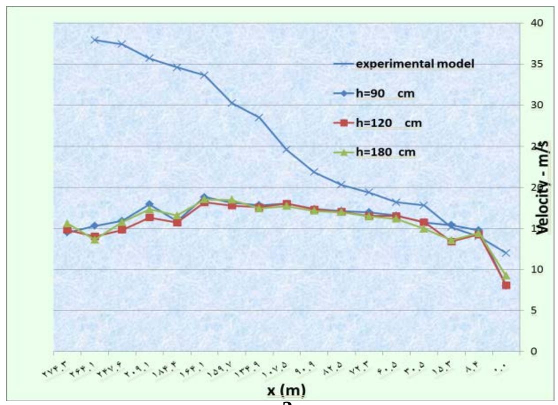

Figure 9. Comparing flow rate in 2290 m^3/s discharge at different heights of steps

As it is seen in the above figures, in all simulated models, the flow rate has decreased from 35 m^3/s to about 15 m^m/s in stepwise model which decreases the probability of occurring cavitation. Furthermore, the depth of flows at the edge of stepped spillways increase compared to similar shoot spillways.

Reducing flow rate and increasing depth, bearing in mind that we would not confront index cavitation of less than 0.6, the probability of cavitation at stepped spillways gets close to zero, while the probability of occurring cavitation in index is less than 0.25.

Discussion

As it is clear, this research aims at numerical modelling of Hydraulic flow at stepped spillways of dam releases, as well as investigating cavitation phenomenon based on FLOW-3D model.

At the beginning of this research, the model were tested with regard to calibration and verification. Because this research aims at investigating dam spillway releases and this has been a shoot one, some models were made by changing shoot into steps. In making this model in FLOW-3D software, the results from previous researchers were used to estimate dimensions of the steps. Discharges which were used in designing shoot spillways of dam releases were considered to help investigating simulation of the model. According to results obtained from this research, the modelled shoot spillway which had been designed for preventing dam releases from cavitation had been the best choice, while changing it into stepped spillways requires its width to increase many times which is not economical at all. From another point of view, stepping up dam spillway releases is the subject of this research, with Debbie flood of 10000 years and less, models were made for simulation and performing researches. Furthermore, optimized dimensions of steps were determined and were investigated based on modelling simulations.

Comparing diagrams of flow rate and flowing depth were determined based on both laboratory results and results from numerical simulations which becomes possible through changing shoot spillways into stepped ones. The flow rate has decreased to large extent which decreases the occurrence of cavitation phenomenon.

At the end, it was concluded that with constant dimensions of spillways by changing shoot spillways into stepped ones, this spillway can answer Debbie flood until 900 m^3/s in case we want Debbie to increase into PMF level and increase width of the spillway into two times.

As it is seen in modelling figures, in some cases we have observed hydraulic jump from steps which can be solved by changing dimensions of them, i.e. by decreasing its amount.

While in this research, climatic part and overflow arc is considered constant, and no step is built on the overflow arc. If we did not consider this limitation and we built step on the overflow arc, we could decease flow rate at the beginning of step so that we could provide a better model for optimizing energy loss.

Conclusion

Verifying results from numerical model indicate that FLOW-3D software can be used for simulating flow on stepped spillways and investigating hydraulic flow.

Stepped spillway is function of the discharge of the width of spillway. Therefore for using stepped spillways at dams, it is suggested that optimized width of spillway for floods of 10000 years and PMF be calculated and based on it stepped spillways be selected. In new designs, using roller compacted concrete (RCC) dams has solved this problem. In many cases of discharges, higher width of spillway is solved.

Increasing the slope of steps in stepped spillways leads to higher energy loss. In other words, in a constant discharge, the more ratio of height to length increases, the more energy loss occurs. In constant dimensions of steps, increasing discharge leads to reduction of energy loss. Also, increasing the slope of spillways would decrease the length of steps, if their height is considered constant.

In all cases, ratio of flow rate to shoot spillway will decrease and considerable increase in cavitation index (more than 0.6) causes cavitation to get close to zero.

References

Abrishami, J., & Hoseini, M (2005). Hydraulic of open canals. Khorasan: Imam Reza University publishers.

Ajdari Moghadam, M., Shah Heydari, H., & Taba'tabaei, J. (2010). Two-phase flow characteristics of stepped spillways (case study: Siah Bisheh Dam). Tehran: University of Tarbiat Modars: Ninth annual conference of Hydraulic.

Ashourian, M. (2012). Investigating the effects of geometric dimensions of stepped spillways at Siah Pisheh dam on energy reduction using numerical models. MA thesis, Tehran: Azad University.

Barani, Q., & Sadri, A. (2003). Energy loss on steep stepped spillways. Isfahan: sixth international conference of civil engineering, Sanati University of Isfahan.

Barani, Q., & Sohrabi Pour, N (2003). Investigating energy loss on edged stepped spillways. Isfahan: sixth international conference of civil engineering, Sanati University of Isfahan.

Birami M. (2006). Constructions for water transfer. Isfahan, Sanati University publishers.

Boyes,K.M., & Donohoo,S.M. (2001). Investigation of Spillway Behaviour under Increased Maximum Flood by Computational Fluid Dynamic Technique, 14th Australasian Fluid Mechanics Conference,10-14 December 2001.

Chanson, H. (1994). Jet flow on stepped spillways, Discussion. J.Hydr.Eng. ASCE , 20.

Chanson, H. (1995). Hydraulic Design of Stepped Cascades, Channels, Weirs and Spillways. Pergamon, Oxford, UK, Jan.

Chanson, H. (1999). The Hydraulics of Open Channel Flows : An Introduction. Edward Arnold, London,UK.

Chanson, H. (2001). The Hydraulics of Stepped Chutes and Spillways. Balkema, Lisse, The Netherlands.

Chanson, H. (2006). Hydraulics of Skimming Flows on Stepped Chutes: the Effects of Inflow Conditions?, J of Hyd. Res., IAHR, 44(1), 51-60.

Chanson, H., & Gonzalez, C.A. (2004). Stepped Spillways for Embankment dams: Review, Progress and Development in Overflow Hydraulics. Proc. Intl Conf. on Hydraulics of Dams and River Structures, Tehran, Iran, Balkema Publ. The Netherlands, pp. 287-294.

Chanson, H., & Tombes, L. (2004). Hydraulics of Stepped Chutes: the Transition Flow. J of Hyd. Res., IAHR, 42 (1), 43-54.

Chanson, H., & Toombes, L. (2002a). Air-Water Flows down Stepped chutes : Turbulence and Flow Structure Observations. Intl J of Multiphase Flow, 27 (11), 1737-1761.

Chanson, H., & Toombes, L. (2002b). Experimental Investigations of Air Entrainment in Transition and Skimming Flows down a Stepped Chute. Can. J of Civil Eng., 29 (1), 145-156.

Choi,W., & Kim,M.H. (2002). Surface Fluid Flows Over Spillway, Journal of hydraulic Engineering.

Chow,V.T. (1959). Design of small dams, McGraw Hill, 377-365.

Esmaeili, A. (2011). Optimizing geometry of stepped spillways suing FLUENT. MA thesis, Tehran: Azad University, Branch of Sciences and Researches.

Falvey,H.T. (1990). Cavitation in Chutes and Spillways, Bureau of Reclamation, Denver CO,USA.

Franc,J.P., & Michel,M. (2006). Fundamentals of Cavitation, Kluwer Academic Publishers.

Ghasem Zadeh, K. (2013). Simulating hydraulic questions in FLOW-3D. Tehran: No Avar publishers.

Ghodsian, M (2006). Hydraulic of dams. Theran: university of Tarbiat Modares.

Gonzalez, C.A. (2005).An Experimental Study of Free-Surface Aeration on Embankment Stepped Chutes. Ph.D. Dissertation, Dept. of Civil Engineering, University of Queensland, Brisbane, Australia.

Gonzalez, C.A., & Chanson, H. (2004). Interactions between Cavity Flow and Main Stream Skimming Flows: an Experimental Study. Can J of Civ. Eng., 31 (1), 33-44.

Henderson, F.M. (1966). Open Channel Flow. MacMillan Company, New York, USA.

Ho,D., Boyes,K., & Cooper,B. (2003). Numerical Flow Analysis For Spillways, 43rd ANCOLD Conference Hobrat,Tasmania.

Horner, M.W. (1969). An analysis of flow on cascades of steps, U.K: University of Birmingham.

Hoseini, M., Kavianpour, M., & Talebi, A. (2010). Investigating effects of slopes, number and places of steps in stepped spillways using numerical models. Tehran: University of Tarbiat Modars: Ninth Annual Conference of Hydraulic.

Jafari Nia, R., & Mousavi Jahromi, H (2005). Evaluating energy loss in stepped spillways in spilling and non-spilling regimes. Kerman University: the 2nd Conference on Watershed and Water and Soil Management.

Jamshidi, M., & Attari, J. (2008). Investigating adequacy of different hydraulic relations between for designing stepped spillways in SiaH Bisheh dam. Sanati University of water and power. Seventh Conference on Hydraulic, Iran.

Kavianpour, M., Hoseini, M., & Roshanov, R. (2009). Investigating features of two-phase flows in stepped spillways. Shiraz University, Eighth International Conference of Civil Engineering.

Khosrojerdi, A. (2010). Hydrodynamics. Tehran: Azad University, Branch of Sciences and Researches.

Majd Zadeh Tabatabaei, M., & Shoraei, H. (2001). Designing stepped spillways, a suitable tool against energy loss. Third Conference of Hydraulic, Tehran University.

Mousavi Jahromi, H., Bina, M., & Salmasi, F. (2005). The effects of geometry of stepped spillways on energy loss. Kerman University: Fifth Conference of Hydraulic, Iran.

Nouri, H. (2013). Investigating cavitation phenomenon in shoot spillway overflows. MA thesis, Kish Azad University, International Branch.

Rajaratnam, N. (1990). Skimming Flow in Stepped Spillways. J of Hyd. Engrg., ASCE, 116 (4), 587-591.

Rostami Ravari, A. (2005). Geometric optimization of Stepped spillways. PhD dissertation, Tehran, Azad University, Branch of Sciences and Researches.

Salmasi, F (2009). The effects of number of steps on energy loss in stepped spillways based on new methods of designing. Journal of Water and Soil Sciences, 19 (1).

Salmasi, F. (2003). Investigating energy loss in laboratory based on stepped spillways. Isfahan: Sixth International Conference of Civil Engineering, Sanati University of Isfahan.

Sar Afraz, M. (2010). Comparing energy loss using experimental relations with numerical models in stepped spillways in Iran. Tehran: University of Tarbiat Modars: Ninth Annual Conference of Hydraulic.

Sar Afraz, M., Attari, J., & Maroufi, N. (2011). Determining place of beginning bleed in stepped spillways with mild slope using numerical models. Gilan: Tenth Conference of Hydraulic, Iran.

Savage,B.M., & Johnson,M.C. (2001). Flow Over Ogee Spillway: Physical And Numerical Model Case Study, Journal of Hydraulic Engineering.

Shafaei Bajestani, M. (2010). Principles and functions of physical-hydraulic models. Ahvaz: Chamran University Publishers.

Shamsaei, A., & Pak Nahal, F. (2005). Shoot hydraulic and stepped spillways. Tehran: Institute of publishers of Sanati Sharif University.

Sorensen, R.M. (1985). Stepped spillways hydraulic model investigation, J.Hydr.Engrg., ASCE ,111.

Versteeg,H.K., & Malalasekera,W. (1995). An Introduction to computational fluid dynamics, Logman Scientific and Technical, New York.

Yasuda, Y., & Ohtsu, I.O. (1999). Flow Resistance of Skimming Flow in Stepped Channels. Proc. 28th IAHR Congress, Graz, Austria.

Yazdani, A. (1998). Investigating the effects of the slope of stepped spillways on energy loss. International Journal of Engineering, 5.