The estimation of investment risk in an asset portfolio by using value at risk method (VAR)

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Abstract

This study attempts to use value at risk method (VAR) as risk measurement criterion in formation of household asset portfolio. To do this, the data which are related to the assets price including: bank deposit, bonds, stock, exchange, coin, and housing that are used in time period of 1991 to 2011 and the value at risk of portfolio is calculated in reliability level of 90%, 95%, and 99% and in time periods of one year and 19 years. After calculating returns, return standard deviation, correlation coefficient among assets and VAR of every asset, the optimal mix of assets is extracted by using variance- mean model and Matlab software and assets portfolio risk is calculated by VAR method. The results indicated that there is the most portfolio risk of 55/13% with the probability of 99% for high risk people and the lowest portfolio risk of Zero%, with the probability of 90% for low risk people. In one year period, there is also the most portfolio risk of 17/51% with the probability of 99% for high risk people and the lowest portfolio risk of 88% with the probability of 90% for low risk people.

Keywords: risk, return, optimal portfolio, value at risk (VAR)

Introduction

Naturally, an individual maintains his assets in the form of a set of alternative asset.For example, in asset portfolio of a person can be pointed to a set which includes cash, bank deposits, durable goods, jewelry, currency, stock and ... each onehas its own special risk. Individual in his decision on how to allocate his assets to each of the mentioned alternative, pays attention to the risk of each of them. Risk is as an important criterion for investment decisions and investors that invest regardless of the risk rate put himself in adverse conditions. Risk should be a little, because the existed risk in asset is one of the determinant factors of intended return's rate of investors. What exist in financing new ideas in relation to the risk refers to unfavorable risks and their measurement.Various measures have been introduced to identify the types of risks by experts. One of these measures is value at risk. In this method, the assets portfolio risk is estimatedfor a certain time horizon and a certain reliability level. In this paper, by the help of "mean - variance" model attributed to Markowitz (mean as a criterion of returns and the variance as a criterion of risk), optimal combination of assetswere extracted through MATLAB software for the time periodof 1991-2011in Iran. Then the risk of asset portfolio is determined by Value at Risk method. For this purpose, first, the theoretical basis of the research was discussed and followed by a few studies in this area. Then the utilized model is introduced and analyzed for conducting this study and the statistical analysis and its results are presented.

Review of Literature

The theoretical concepts of Value at Risk models (VaR)

Family Value at Risk belongs to the family of undesirable risk measurement criteria (Matrix Company of Systems Analysts). This index as a statistical criterion calculates the maximum expected loss of an asset or portfolio at a specified time periodand with a certain probability and reports quantitatively. In other words, the value at risk specifies the amount of asset or portfolio value that is expected to lose over a specified time period and with certain amount of probability (ibid.). Value at risk is a probable phrase that expresses the possible change in portfolio value due to the market factors

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change within a certain period of time. This criterion does not explain about this that, the actual loss to what extent will be more than the probable amount. Risk calculation in today's investment portfolio includes a variety of financial instruments such as stocks, bonds andtypes of derivative instrumentsthat is measurable only through this indicator. Other methods cannot be used to calculate risk because of the special characteristics of derivative instruments, such as no linear relationship between the returns tools and the main committed assets. Maximum possible loss of a portfolio is measured due to the damage density function which is displayed with F. In fact, the Value at Risk is the quantile of function F at critical levels (α =10%, 5% and 1%) thus, based on the definition:

$$P(loss \leq VaR) = \int_0^{var} f(l)dl = 1-\alpha \qquad (1)$$

If we have had the distribution of asset returns, based on the definition:

$$P(r \leq VaR_r) = \int_0^{var} f(r)dr = 1-\alpha \qquad (2)$$

$$VaR_r = f^{-1}(1-\alpha) \qquad (3)$$

In the above equation, f (r) is the probability distribution function of portfolio changes rate. ris equal to the rate of critical changes and in fact, is the quantile of reliability level (1-α).

If the distribution of the assets' portfolio value to be specified, the Value at Risk is not difficult to calculate. Solely for convenience of calculation, the normal distribution is used to represent the distribution of the future value of portfolio. The value at risk calculation in intended reliability level and assuming that, the distribution isnormal, was done through the following relationship:

$$Var = p_0 - F^{-1}_p(\alpha) = P_0 - P^c \qquad (4)$$

$$var = P_0 - (\mu - Z \times \delta) \qquad (5)$$

In which:

P^c: the critical value of the portfolio

Z_x: the inverse amount of the standard normal cumulative distribution function at the reliability level of α-1.

μ: the mean of the portfolio value

δ: standard deviation of the portfolio value

Obviously, we will never have access to the actual parameters of the distribution of portfolio value, but estimations of them are available. These estimates are and S that are used instead of μ and δ respectively.

Reliability level and time horizon

The utilization of VaR requires optional choice of two parameters. These parameters include:

1. Reliability level

2. Time horizon (Maintenance period) (Rai & Saidi)

Determining the level of reliability depends on the subject's objectives. To determine the range of risk, the reliability levels are usually chosen in the range of 95% to 99%, because these reliability levels create a few violations and enforce investors to consider the range seriously. When the VaR is used for reporting or comparison, probably the reliability level is determined in a way that to be comparable to other institutions. Also, in this case, the reliability level is selected in the range of 95% to 99%.

Routine maintenance periods, are usually a day, a week, ten days or a month. But institutions can also choose other maintenance periods. Depending on the investment or reporting horizon the maintenance periods of three months or more can also be used. The maintenance periods also depends on the liquidity of the markets in which the institution operates.

The calculating method of Value at Risk

Considering the normal assumption for distribution of assets returns is very common and reasonable. Also, this distribution has been taken in to consideration from this aspect that is described only by two parameters. By assuming the normal distribution of returns, the Value at Risk is calculated as follows (Matrix Company of Systems Analysts).

$$VaR_t = -P_{t-1}(\mu_t - \delta_t Z_\alpha) \qquad (6)$$

VaR_t: Value at risk for future periods

P_{t-1}: Current share prices

μ_t: the average of returns in period t

δ_t: the standard deviation of returns in period t

Z_α: the amount of the standard normal variable at a reliable level of α-1

$\mu_t - \delta_t Z_\alpha$: percent of the value at risk (VaR%) with a negative sign indicates that,multiplying it in the negative current price produce the VaR

One of the best features of parametric approaches is to make possible the estimate of the Value at Risk at any reliability level and ateach intended maintenance

period. In normal distribution, the intended reliability level is reflected in Z_α and maintenance period appears in μ_t and δ_t. If μ_t and δ_t to be respectively the mean and standard deviation of returns over a specific time period. (e.g., a day), the mean and standard deviation of returns relevant to the number of h period of this certain period are obtained from the following relation:

$$\mu(h) = h(\mu) \tag{7}$$

$$\delta^2(h) = h\delta^2 \rightarrow \delta(h) = \sqrt{h}\delta \tag{8}$$

By substituting these relations in the one periodic VaR equation, the multi periodic VaR obtains over the h period of time and alpha error level with assuming the temporal autonomy of returns:

$$VaR_{ht} = -p_{t-1}(h\mu_t - \sqrt{h}\delta Z_\alpha) \tag{9}$$

Benefits of the Value at Risk

1. Value at Risk can be applied about the portfolios consists of stocks, bonds, goods and derivative instruments.

2. One of the advantages of the value at risk compared with the traditional criteria of risk like beta and Diresh is that, reflects both components of the risk, while the beta coefficient and Diresh consider only one component. VaR also consists of the unreliability related to the risk factor and thus reflects both risk components.

3. Value at Risk gives insights to investors about the nature and types of risk. Applying restrictions for assets allocation through VaR causes portfolio managers lead investment risk to those parts that have higher potential for returns and diversify their activities based on the risk reduction strategy.

4. The feature of Value at Risk is that looks forward. Namely, estimates the total risk of the existed portfolio for future periods. Therefore, VaR as a looking forward risk measurement criterion can provide useful information about the expected portfolio risk in the future.

5. Value at risk empowers us to assemble the secondary opportunities' risk to the total risk of portfolio and by doing so takes into account the interactions and correlations among the different risk factors and this is another VaR attractiveness, because most risk measures do not provide the possibility of evident aggregation of partial risks.

6. Simple concept of value leads to its increasingly attractiveness. Understanding the other risk measures such as standard deviation, beta coefficients andetcaccording to their definitions is difficult for most investors. But the simple definition of risk includes more understandability for investors and market participants.

Review of previous studies

Hanifi (2001) in his doctoral thesis in addition to introducing the criterion of Value at Risk, has examined the risk taking rate ofthe accepted companies inIran Stock Exchange and come to this conclusion that, the risk of financial index is greater than the industry index. He also compared the manufacturing companies' risk taking with investment companies and concluded that, the risk of selected investment companies is more than the manufacturing companies.

Khalouzadeh and Amiri (2006), based on the Value at Risk theory, have determined the optimal stock portfolio in stock market of Iran. In their study, the optimal stock portfolio is obtained through the Genetic Algorithm which has the maximum profit and is considered as the criterion of VaR estimate.

Rahmati (2008) selects the portfolio in exchange market of Iran by approach based on the value at risk and by using genetic algorithm. This model that has been applied in a small and limited market, indicates that, the criterion of value at risk in exchange market is effective than Markowitz criterion.

Khalili (2008) estimates the market risk of an investment portfolio based on the value at risk. The results show that 7% of the investment portfolio is in danger. Since this index determines the amount of capital at risk of company at certain reliability level, is better than other methods for decision makers of company to optimize the portfolio.

Eghbal Niya (2005) calculates the Value at Risk by using the cash return index but the optimal portfolio has not been determined.

Lotfaliei (2005) calculates the stock portfolio risk of Mine and Industry Bank by using criterion of Value at Risk (Abbasi, E. 2008).

Perignon et al (2007) based on the data from the six largest Canadian commercial banks, determined the investment risk in them through Value at Risk criterion (I think source is wrong please check it)

Koierkand Mao in their studies showed thatthe investors in terms of behavior and individual calculations have focused more on the unfavorable risk than the risk that includes positive and negative fluctuations (Yamaiand Yoshiba, 2002).

The statistical population, sample and methodology

The statistical population of this study consists of assets that are maintained in households' asset portfolio. Since access to the price of all assets that constitute households' asset portfolio over a period of 20 years is impossible, therefore, the assets such as stocks, bonds, exchange, bank deposits, coins and housing that access to their price is possible were studied. The necessary information to conduct research is the price of mentioned assets during 21 years (1991-2011). This information can be extracted from the center Bank's website. To conduct this research, the following steps are done.

1. Calculate the expected return and risk of assets portfolio by Markowitz model

2. Calculate the risk of assets portfolio by value at risk model

3. Calculate the correlation coefficient between the assets portfolio returns

2. Solving the quadratic liner programming model with MATLAB software and determining the optimal weight of assets in the portfolio

3. Calculation of investment risk of assets portfolio by Value at Risk Model

The model used in the study

In this paper, the Markowitz model has been used to select the optimal assets portfolio and the value at risk model to estimate the assets risk. The Markowitz model is a quadratic programming model which is based on the mean and variance of assets returns and assumption of normal distribution of assets returns is its main assumptions. Suppose that, we have the portfolio of n different assets whose returns have the multivariate normal distribution with mean μ and matrix of variance - covariance to be sigma. μ is a vector of n × 1 and sigma is an n × n matrix that the variances are located in the main diagonal and covariances in other places of matrix. The W row vector was considered as the weights of each asset in the portfolio consider. Dimension of this vector is 1 × n. Thus, the mean and variance of the portfoliowere obtained through the following relationship.

$$\mu_p = w\mu \qquad (10)$$

$$\delta^2 p = w\sum w^t \qquad (11)$$

The aim of the investment is to minimize the risk of portfolio at thecertain expected return. So we have:

Min: δ_p^2

s.t:

$$1. E(R_p) = \sum w_i E(R_i)$$

$$2. \sum_{i=1}^{k} w_i = 1$$

$$3. w_i \geq 0$$

In this equation:

Standard deviation of portfolio returns is calculated from the following equation:

$$\delta_p^2 = \sum_{i=1}^{n} w_i^2 . \delta^2(R_i) + \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j cov(R_i, R_j)$$

$$\rightarrow \delta_{p=}\sqrt{\sum_{i=1}^{n} w_i^2 \delta^2(R_i) + \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j cov(R_i, R_j)} \quad (12)$$

$E(R_p)$: Expected returns of the portfolio
$E(R_i)$: Expected returns of the ith assets
w_i: The weight of each asset in the portfolio
$cov(R_i, R_j)$: Covariance of returns on i[th] and j[th] assets

In case of normal distribution of assets returns, value at risk is a linear function of the mean and standard deviation of returns (Matrix Company of Systems Analysts, 2009).

Standard deviation of one period

$$VaR_t = -p_{t-1}(\mu_t - \delta_t Z_\alpha) \qquad (13)$$

In which μ_t and δ_t are respectively mean and standard deviation of the asset returns or portfolio and Z_α is the amount of standard normal test statisticat the significance level alpha. This approach is to calculate the value at risk of assets. Of course this does not mean that the calculation of value at risk for portfolio is different from individual assets, but their only difference is that, the parameters associated with assets portfolio to be replaced with individual assets. However, calculation of value at risk at the portfolio level is more complex than individual assets. In many cases, to calculate the value at risk of the portfolio put this assumption as a base of our work that,the returns on individual assets has multivariate normal distribution and this assumptionis equal to consideringnormal distribution to portfolio returns.

If the current price of the portfolio to be shown with p_{t-1}, the value at risk during the h maintenance period and reliability level of 1-α is equal to:

$$VaR_{ht} = -p_{t-1}\left(h\mu_{p,t} - \sqrt{h}\delta_{p,t}Z_\alpha\right) \qquad (14)$$

Data analysis

To select the optimal portfolio, the annual returns of assets were calculated over the time period of 1991-2011 and entered into the MAT-LAB software. Then, the variance-covariance matrix, correlation coefficients among the assets and the expected returns on assets were calculated. Following tables show the correlation coefficients, variance-covariance matrixand returns on assets.

Table 1. Correlation coefficients among the asset returns

Exchange	Housing	Bonds	Gold coin	Bank deposit	Stock	
Housing	1	0.04	0.08	0.04	0.14	- 0.04
Bonds	0.04	1	0.21	0.48	0.20	0.70
Gold coin	0.07	0.21	1	- 0.23	0.47	0.67
Bank deposit	0.03	0.48	- 0.23	1	0.25	0.25
Stock	0.14	0.20	0.47	0.25	1	0.38
Exchange	- 0.04	0.70	0.67	0.25	0.38	1

Source: Calculations of the researcher

Figures contained in the table shows the relationship among the assets return in the asset portfolio. The more negative correlation coefficient indicates the better alternative of assets in the asset portfolio. Variance-covariance matrix is extracted by using a series of annual returns of assets as follows.

$$V = \begin{pmatrix}
414.51 & 1.28 & 32 & 0.88 & 129.84 & -15.97 \\
1.28 & 2.39 & 6.67 & 0.80 & 13.59 & 18 \\
32 & 6.67 & 422.56 & -5.18 & 423.40 & 228.76 \\
0.88 & 0.80 & -5.18 & 1.18 & 11.98 & 4.58 \\
129.84 & 13.59 & 423.40 & 11.98 & 1.94 & 276.73 \\
-15.97 & 18 & 28.76 & 4.58 & 276.73 & 73272.
\end{pmatrix}$$

In which the main diagonal entries indicate the variance of returns of each assets and other entries represent the covariance among the assets returns. Therefore, the problem of minimum finding of the mean - variance model is as follows:

$$\text{Min} \, 414/51w_1^2 + 2(1/28)w_1w_2 + 2 \times 32w_1w_3 + 2 \times (0/88)w_1w_4 + 2$$
$$\times (129/84)w_1w_5 + 2(15/97)w_1w_6 + 2/39w_2^2 + 2(6/6)w_2w_3$$
$$+ 2(0/8)w_2w_4 + 2(13/59)w_2w_5 + 2 \times 18w_2w_6 + 422/56w_3^2$$
$$- 2(5/18)w_3w_4 + 2(423/40)w_3w_5 + 2(228/76)w_3w_6 + (1/18)w_4^2$$
$$+ 2(11/98)w_4w_5 + 2(4/58)w_4w_6 + 1/94w_5^2 + 2(276/73)w_5w_6 + (272/73)w_6^2$$

s.t:

1) $20/5w_1 + 17/66w_2 + 20/81w_3 + 17/13w_4 + 29/5w_5 + 12/46w_6 = \infty$

2) $w_1 + w_2 + w_3 + w_4 + w_5w_6 = 1$

3) $w_{i \geq 0, i=1,2,\ldots,6}$

In the above equations are respectively share of housing, bonds, gold coins, bank deposits, stocks, and exchange in portfolio and α is the expected return of portfolio.

With solving the above problem based on the w, the optimal assets weights in the portfolio will be achieved.

To estimate the optimal assets portfolio for low-risk, medium risk and high risk individuals, 60 optimal portfolios were specified and the optimal weight of assets, the expected returns and risk of the optimal portfolio were calculated through the information entered.

Table 2. Share of assets in the optimal portfolio

Assets Risk Taking	Exchange	Stocks	Bank Deposits	Coins	Bonds	Housing
low-risk	0	0.10	0.09	0.03	0.68	0.10
medium risk	0	0.37	0	0.12	0.17	0.34
high risk	0	0.78	0	0	0	0.22

Source: Calculations of researcher

The above table shows that, with regard to the risk and assets returns for low-risk individuals the share of bonds in their asset portfolios is the greatest and the share of exchange in their asset portfolio is the lowest. But the housing and stock constitute the combined asset of risky individuals that is the biggest share of stock. Figures of Table show that, enhancement of individuals' risk taking increase the housing and stock share in their combined assets and decrease the share of bank deposits and bonds. After determining the share of assets in optimal portfolio, the following tables show the Value at Risk at the reliability levels and different time horizons and also the necessary information to calculate it.

Estimating the Value at Risk of assets

Then, the optimal portfolio risk was calculated through the value at risk method, at reliability levels of 90%, 95% and 99% and in time periods of one year and 19 years.

The following Table shows the value at risk of assets at different levels of reliability and at one-year period.

Table 3. The value at risk percentage of assets at one year time horizon

Assets Significance Level	Exchange	Stocks	Bank Deposits	Coins	Bonds	Housing
$\infty = 0.1$	4.19	11.42	0	4.94	0	4.9
$\infty = 0.05$	5.57	15.09	0	6.65	0	6.6
$\infty = 0.01$	8.15	21.98	0	9.86	0	9.78

Source: Calculations of the researcher

According to the above table, at all levels of reliability the highest risk associated with stocks and the lowest related to bonds and bank deposits. In explanation of calculated numbers for assets risk and for example, for the stock can be said: respectively with probability of 90%, 95% and 99% during the next year, the loss caused by stock keeping won't be more than 11.42, 15.09 and 21.98 percent respectively. In one-year time horizon, the maximum loss of bank deposits and bonds will be zero percent at different levels of reliability.

Estimating the Value at Risk of the optimal portfolio

The value at risk of optimal portfolio were calculated at reliability levels of 90%, 95% and 99% and at one- year and 19-year time horizon. Following table show the Value at Risk of optimal portfolio.

Also, the value at riskin the1-year time horizon is based on the following table.

Based on the statistical analysis results and studies conducted can be stated:

1. With probability of 99 % and at 19-year time horizon, the maximum loss of portfolio for low risk, medium risk and high risk are respectively, 1.22, 20.31 and 55.13 percent.

2. The maximum loss of portfolio for low risk, medium risk and high risk are respectively, 0, 9.28 and 30.91 percent, with probability of 99 % at 19-year time horizon.

Table 4. Value at risk in 19-year time horizon

Risk taking	The mean of returns	SD of portfolio	time horizon	Z 0.1	VaR	Z 0.05	VaR	Z 0.1	VaR
Low-risk	19.21	6.01	19 years	2.326	1.22	1.645	0	1.282	0
medium risk	23.38	19.85	19 years	2.326	20.31	1.645	9.28	1.282	2.94
High risk	27.54	35.54	19 years	2.326	55.13	1.645	30.9	1.282	18
Totalsubjects	23.37	20.46	19 years	2.326	25.5	1.645	13.4	1.282	6.98

Source: Calculations of the researcher

Table5.Value at Risk at the 1-year time horizon

Risk taking	The mean of portfolio returns	SD of portfolio returns	time horizon	Z 0.1	VaR	Z 0.05	VaR	Z 0.1	VaR
Low-risk	0.97	1.38	1 year	2.326	2.24	1.645	1.35	1.282	0.88
medium risk	1.23	4.55	1 year	2.326	9.36	1.645	6.27	1.282	4.61
High risk	1.45	8.15	1 year	2.326	17.51	1.645	11.96	1.282	8.99
Totalsubjects	1.22	4.69	1 year	2.326	9.7	1.645	6.52	1.282	4.82

Source: Calculations of the researcher

3. In 19-year time horizon, the maximum loss of portfolio for low risk, medium risk and high risk are respectively 0, 2.94 and 18 percent, with the probability of 90 %.

4. With probability of 99 % and at 1-year time horizon, the maximum loss of portfolio for low risk, medium risk and high risk are respectively 2.24, 9.36 and 17.51 percent.

5. In 1-year time horizon, the maximum loss of portfolio for low risk, medium risk and high risk are respectively 1.35, 6.27 and 11.96 percent, with probability of 95 %.

6. The maximum loss of portfolio for low risk, medium risk and high risk are respectively 0.88, 4.61 and 8.99 percent, with probability of 90 % at 1-year time horizon.

7. In 19-year time horizon, the mentioned maximum loss of portfolio for low risk, medium risk and high risk with probabilities of 99%, 95% and 90% are respectively 25.55, 13.4 and 6.98 percent.

8. With probabilities of 99%, 95% and 90% and in 1-year time horizon, the mentioned maximum loss of portfolio for low risk, medium risk and high risk are respectively 9.7, 6.52 and 4.82 percent.

Conclusions and Recommendations

1. In a 19-year time horizon, the greatest risk of portfolio is 55.13% with 99% probability for high risk subjects and the lowest is 0 % with probability of 90% for low risk subjects.

2. In a 1-year time horizon, for high risk individuals, the greatest risk of portfolio is 17.51% with 99% probability and for low risk individual the lowest risk is 0.88% with probability of 90%.

3. Due to the efficiency of Value at Risk method in calculation of risk, it is suggested to researchers to calculate the risk of other portfolio types.

References

Abbasi, A.A. (2007). *Assessment and comparison of performance of different methods of the value at risk calculating in exchange market of Iran,* MA thesis, Faculty of Management, Sanati Sharif University.

Abbasi, E. (2008).The use of value at risk in formation of optimized stock portfolio in Tehran Stock Exchange. *Economic Research Journal, 87,* 75-90.

Anderson, B. (2002). *Portfolio allocation over the life cycle: Evidence from Swedish household data,* Uppsala University, Department of Economics in its Series Working Paper Series.

Barone-Adesi, G., & Giannopoulos, K. (1996). A simplified approach to the conditional estimation of Value at Risk (VAR), Working Paper, Faculty of Business, Alberta, Canada.

Bekkers, B., Doeswijk, R.Q., &Lam, T.W. (2009). Strategic asset allocation: Determining the optimal portfolio with ten asset classes, *The Journal of Wealth Management, 12*(3), 61.

Best, M.J. & Grauer, R.R. (1991).On the sensitivity of mean−variance-efficient portfolios to changes in assetmeans: Some analytical and computational results. *Review of Financial Studies, 4*(2), 315−342.

Best, P. (1998). *Implementing value at risk*, England: Johnwhiley and Sons, vol.1.

Chan, L.K.C., Karceski, J. & Lakonishok, J. (1999). On portfolio optimization: Forecasting covariances and choosing the risk model. *Review of Financial Studies, 12*(5), 937−974.

Chen, J. (2000). *Where is the efficient frontier?* University of Northern British Columbia - School of Business Working Paper Series Available at SSRN: http://ssrn.com/abstract=239048 or doi:10.2139/ssrn.239048.

Chen, W., Chung, H., Ho, K., & Hsu, T. (2003). *Portfolio optimization models and mean- variance spanning test,* Working Paper Series.

Cocco, J. (2000). Portfolio choice in the presence of housing, *The Review of Financial Studies, 18*(2), 535-567.

Hagin, R.L. (2004). *Investment management-portfolio diversification, risk, and timing-fact and fiction,* Hoboken, New Jersey: John Wiley & Sons, Inc.

Hanifi, F.(2001). *Investigating the risk taking rate of accepted companies in Tehran Stock Exchange through the measure of value at risk, PhD dissertation, Science and Research University of Tehran*

Jobson, J., & Korkie, R. (1980).Estimation for Markowitz Efficient Portfoli, *Journal of the American Statistical Association*, *1*(75), 544−554.

Jones, C. P. (2012), Investment Management, Translator: Tehrani, R. Nourbakhsh, A., Negha-he Danesh Publications, Eighth Edition

Jorion P. (2000). *Value at risk: The new benchmark for managing financial risk*, Second Edition, New York: McGraw Hill.

Keshavarzian Peyvasti, A. (2007). Estimation of optimal bank interest rates in order to maximize economic growth in Iran (through simultaneous equations system), *Planning and Budget Journal, 108,* 3-30

Khalouzadeh, H., & Amiri, N. (2006).Determining the optimal stock portfolio in   stock exchange market of Iran, based on the value at risk theory. *Economic studies, 73,* 211-231.

Letzelter, J. (2005). Finding the efficient frontier: Power plant portfolio assessment, *Proceedings of Crystal Ball Conference.*

Markowitz, H. (1952). Portfolio selection, *The Journal of Finance, 7.*

Matrix Company of Systems Analysts (2009).*Market risk management, value at risk approach,* Ayande Publications, First Edition.

Mohammad Estakhri, N. (2006). *Selection of a stock portfolio among the shares of accepted companies in Tehran stock exchange by using genetic algorithm optimization model*, MA Thesis, Tehran University.

Perignon, C., Deng, Y.Z., & Wang, Z.J. (2007). Do banks overstates their Value At Risk?, *Journal of Banking and Finance, 32*(5), 783-794.

Raei, R. (1998). *Designing a mathematical model for optimal portfolio selection through fuzzy logic programming*, PhD dissertation, University of Tehran.

Rahmati, M. (1999).*The optimal portfolio selection based on the value at risk as the risk criterion and by using the meta-heuristic algorithms*, M.A. Thesis, Tehran University.

Rai, R., & Saidi, A (2011), *Principles offinancial engineeringand risk management, SAMT publications, Sixth Edition.*

Saranj, A.R. (2007). Problem of portfolio selection by using three criteria: average returns, standard deviation of returns and liquidity in Tehran Stock Exchange, M.A. thesis, Faculty of Management, Tehran University.

Shahrabadi, A., & Bashiri, N. (2010). *Investment Management at Stock Exchange, informing company and stock services,* First Edition.

Sharpe, W. F. (1967). Linear programming algorithms for mutualfund portfolio selection, *Management Science Journal, 13*(7), 449-510.

Sharpe, William F., Alexander, Gordon, J. & Jeffrey, D. Billy (2011). *Investment Management* (M.ShariatPanahi, and A.Jafari, Trans,), Etehad Publications, Third Edition.

Yamai, Y. &Yoshiba, T. (2002). On the validity of Value-at-Risk: Comparative analyses with expected shortfall*, Monetary and Economic Studies, 20,* 57-85.