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Calculation of Asymmetry factor using the Solution of Equilibrium Problem

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Abstract

In this work we presented a theoretical calculation of the asymmetry factor (Shafranov parameter) by solving the simplest Grad–Shafranov Equation (GSE) with the Solove'v assumption for a low-beta and large-aspect-ratio tokamak with a circular cross section. In this paper we calculated the current-independent relation for the asymmetry factor (Shafranov parameter).

Keywords: Shafranov parameter, Solove'v assumption, Tokamak

Introduction

In ohmically heated tokamaks (Salar Elahi et al., 2009; Salar Elahi et al., 2010; Paknezhad et al., 2013) the radial pressure balance is achieved by a poloidal field, and the toroidal force balance is achieved by the equality between the external vertical field force (Lorentz force) and the outward forces due to the toroidal configuration. However, in the toroidal force balance problem, the two opposite forces may be not equal; therefore, the plasma intend to shift inward or outward, Asif et al., 2005, which is dangerous for the tokamak plasma, Salar Elahi et al., 2009. Therefore, the plasma equilibrium study is one of the fundamental problems of magnetically confined plasmas. One of the main challenges of tokamak plasma equilibrium studies is the discovery of parameters that affects the plasma equilibrium states. The asymmetry factor (Shafranov parameter) is one of the main parameters of the tokamaks plasma. From this parameter, much of the tokamak plasma informations, such as the Shafranov shift, poloidal beta, internal inductance, current density profile, plasma pressure, and energy confinement time can be determined (Salar Elahi et al., 2009; Salar Elahi et al., 2010; Paknezhad et al., 2013). As we know, the measurement of the asymmetry factor (Shafranov parameter) is essential in tokamak plasma experiments. In the previous studies (Salar Elahi et al., 2009; Salar Elahi et al., 2010; Paknezhad et al., 2013; Mukhovatov & Shafranov, 1971; Shen et al., 2007), magnetic probes, poloidal flux loops, and diamagnetic loops were used to determine the asymmetry factor for tokamaks. For this reason, array of magnetic probes, flux loops, and diamagnetic loop with a compensation coil were designed, constructed, and installed on the outer surface of the tokamak chamber, and then the asymmetry factor was measured (Salar Elahi et al., 2009; Salar Elahi et al., 2010; Paknezhad et al., 2013; Mukhovatov & Shafranov, 1971; Shen et al., 2007). In this work we presented a theoretical calculation of the asymmetry factor (Shafranov parameter) by solving the simplest Grad–Shafranov Equation (GSE) with the Solove'v assumption for a low-beta and large-aspect-ratio tokamak with a circular cross section, Shen et al., 2007, (Table 1). In this paper, we calculated the current-independent relation for the asymmetry factor (Shafranov parameter).

Table 1: Parameters for a low-beta and large-aspect-ratio tokamak with a circular cross-section.

Parameter	Value
Major Radius	1.22 m
Minor Radius	0.27 m
Toroidal Field	1- 2.5 T
Plasma Current	$100-250kA$
Discharge Time	$\sim 300s$
Electron Density	$1-6\times10^{19}m^{-3}$

Solution of Equilibrium Problem

The simplest solution (Zheng et al., 1996; Asif, 2013), can be found by assuming that,

$$\mu_0(\gamma-1)\frac{\partial u}{\partial\psi}=-A_1,\ F\frac{\partial F}{\partial\psi}=A_2\ , \tag{1}$$

where A_1 and A_2 are constant. This obviously reduces the set of possible current density profile shapes to $J_\phi \propto RA_1-{A_2}/{R}$.

If the plasma is assumed to be up–down symmetric, its shape can be described by four parameters. The equatorial innermost and outermost points, R_i and R_0, and the coordinates of the highest point, (R_t,Z_t) or equivalently, the major radius $R_m={(R_i+R_0)}/{2}$, $R_m\neq R_0$, the minor radius $a={(R_0-R_i)}/{2}$, the elongation $\kappa_0={Z_t}/{a}$, and triangularity $\delta={(R_0-R_t)}/{2}$.

The simplest solution (Zheng et al., 1996; Asif, 2013), is given by

$$\psi=c_1+c_2R^2+c_3(R^4-4R^2Z^2)+c_4(R^2\ln(R)-Z^2)-\frac{A_1}{8}R^4-\frac{A_2}{2}Z^2. \tag{2}$$

To determine these six coefficients, it is necessary to have six equations. We assume that the internal energy vanishes at the boundary, hence $\psi(R,Z)\big|_b=0$ (Zheng et al., 1996; Asif, 2013).

With Eq. (2), the boundary conditions $R=R_0\pm a$, $Z=0$ and $R=R_t$, $Z=Z_t$ gives the following equations:

$$\psi(R_i,0)=c_1+c_2R_i^{\ 2}+c_3R_i^{\ 4}+c_4R_i^{\ 2}\ln(R_i)-\frac{A_1}{8}R_i^{\ 4}=\psi_{180}, \tag{3}$$

$$\psi(R_0,0)=c_1+c_2R_0^{\ 2}+c_3R_0^{\ 4}+c_4R_0^{\ 2}\ln(R_0)-\frac{A_1}{8}R_0^{\ 4}=\psi_0, \tag{4}$$

$$\psi(R_t,Z_t)=c_1+c_2R_t^{\ 2}+c_3(R_t^{\ 4}-4R_t^{\ 2}Z_t^{\ 2})+c_4(R_t^{\ 2}\ln(R_t)-Z_t^{\ 2})-\frac{A_1}{8}R_t^{\ 4}-\frac{A_2}{2}Z_t^{\ 2}=\psi_{90}, \tag{5}$$

We also assume that the plasma is enclosed in a perfectly conducting toroidal boundary with a circular cross section, with radius a, so the normal component of the magnetic field.

$$\frac{1}{R}\frac{d\psi(R_t,z_t)}{dR}=2c_2+4c_3(R_t^{\ 2}-2Z_t^{\ 2})+c_4(2\ln(R_t)+1)-\frac{A_1}{2}R_t^{\ 2}=B_Z(R_t,Z_t), \tag{6}$$

The plasma current can be clearly measured by Rogowski coil, Shen et al., 2007, so the plasma current can be written.

$$2\pi\mu_0 I_p=\iint(RA_1+{A_2}/{2})dRdZ, \tag{7}$$

It is simpler to first solve for a plasma with unit current and unit major radius and use the scaling relations described above to find the final desired equilibrium. However, even in this simplest case only numerical solutions to Eqs. (3-8) have been found. The coefficients can be computed numerically, given a desired plasma description.

We also selected the constraint $\beta_p + {l_i}/{2}$, Shen et al., 2007, because the parameter can be experimentally deduced using discrete magnetic probes, Shen et al., 2007, for a circular cross section tokamaks (Salar Elahi et al., 2009; Salar Elahi et al., 2010; Paknezhad et al., 2013; Mukhovatov & Shafranov, 1971; Shen et al., 2007),

$$\beta_p + {l_i}/{2} = \frac{\left(\oint dl\right)^2}{\left(2\pi\mu_0 I_p\right)^2 \iint RdRdZ} \times \left(2.5A_1 \iint \psi(R,Z) RdRdZ + 0.5A_2 \iint \frac{\psi(R,Z)}{R} dRdZ\right) \quad (8)$$

Asymmetry factor using the Solution of Equilibrium Problem

We have obtained the six coefficient by solving six algebraic equation for a circular cross section tokamak, Shen et al., 2007 , (Table 1).

$$c_1 = -0.004A_1 - 0.034A_2 + 0.23B_Z - 0.359\psi_0 + 2.359\psi_{180} - 1.51\psi_{90} \quad , \quad (9)$$

$$c_2 = 0.034A_1 - 0.084A_2 + 3.823B_Z - 8.359\psi_0 + 12.309\psi_{180} - 6.351\psi_{90} \quad , \quad (10)$$

$$c_3 = 0.024A_1 + 0.024A_2 - 3.553B_Z + 20.359\psi_0 - 30.212\psi_{180} + 3.351\psi_{90} \quad , \quad (11)$$

$$c_4 = 0.012A_1 - 0.1324A_2 + 2.11B_Z - 10.3\psi_0 + 31.212\psi_{180} - 20.311\psi_{90} \quad , \quad (12)$$

$$A_1 = 165.345\mu_0 I_P - 3.10A_2 \quad , \quad (13)$$

$$A_2 = 4105.245\{0.0034B_Z + 0.0061\mu_0 I_P + 0.1049\psi_0 + 0.0646\psi_{180} + 0.151\psi_{90} \pm$$
$$((-0.0034B_Z - 0.0061\mu_0 I_P + 0.1049\psi_0 + 0.0646\psi_{180} + 0.151\psi_{90})^2$$
$$+ 0.0001\mu_0 I_P (0.003B_Z + 0.0361\mu_0 I_P - 1.304(\beta_p + \frac{l_i}{2})\mu_0 I_P + 0.534\psi_0 + 0.446\psi_{180} \quad (14)$$
$$+ 1.151\psi_{90}))^{0.5}\},$$

The parameter A2 has two values, but one value that can be obtained by experimental data is acceptable. The acceptable sign is minus. By using the coefficient in equation (2), the asymmetry factor (Shafranov parameter), can be obtained from (8) as shown in Figure 1. Experimentally the the asymmetry factor (Shafranov parameter), $\Lambda = \beta_p + {l_i}/{2} - 1$, can be measured as follow (Salar Elahi et al., 2009; Salar Elahi et al., 2010; Paknezhad et al., 2013; Mukhovatov & Shafranov, 1971; Shen et al., 2007),

$$\Lambda = \beta_p + \frac{l_i}{2} - 1 = \ln\frac{a}{b} + \frac{\pi R_0}{\mu_0 I_0}\left(\langle B_\theta \rangle + \langle B_n \rangle\right), \quad (14)$$

Where

$$\langle B_\theta \rangle = B_\theta(\theta = 0) - B_\theta(\theta = \pi),$$

$$\langle B_n \rangle = B_n(\theta = \frac{\pi}{2}) - B_n(\theta = \frac{3\pi}{2}), \quad (15)$$

We measured these local magnetic fields with magnetic probes, Shen et al., 2007 at above angles.

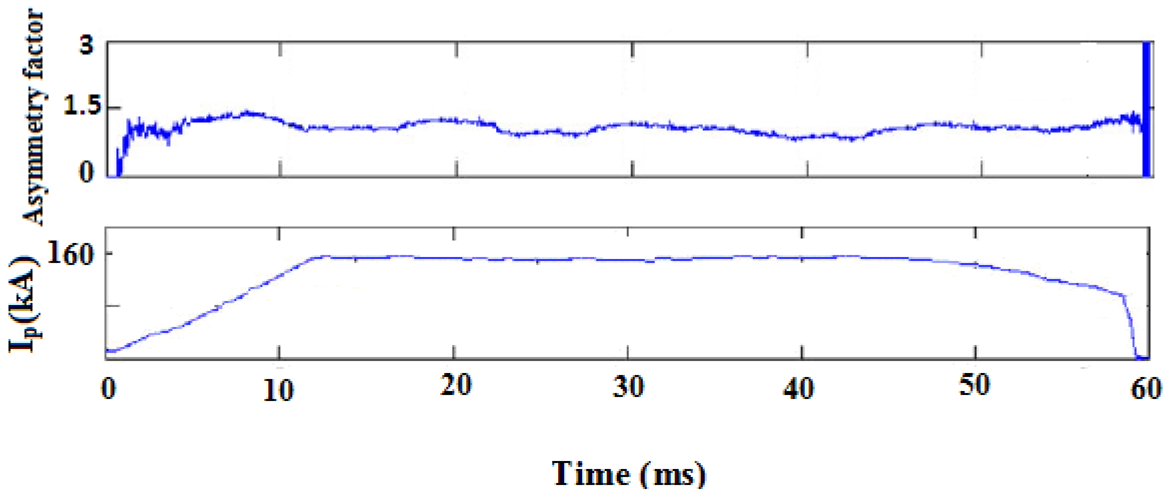

Figure 1: The calculated time evolution of the asymmetry factor (Shafranov parameter) for a low-beta and large-aspect-ratio tokamak with a circular cross section.

Conclusion

In tokamak plasmas equilibrium study, to determine the asymmetry factor (Shafranov parameter) is essential. We presented a theoretical calculation of the asymmetry factor (Shafranov parameter) by solving the simplest Grad–Shafranov Equation (GSE) with the Solove'v assumption for a low- beta and large-aspect-ratio tokamak with a circular cross section. In this paper we calculated the current-independent relation for the asymmetry factor (Shafranov parameter).

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