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Multi-item Economic Order Quantity with Consideration of Transportation Costs and Vehicle Capacity

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Abstract

In this paper, a supplier-retailer logistic system for multi-item is investigated as a two-echelon environment. There is a single location in each echelon; the unique supplier at the first echelon has to replenish the retailer's warehouse at the second echelon. A model minimizing the total cost of purchasing, ordering, inventory, variable and fixed transportation costs is developed. Multistage shipment with a specific number of vehicles is allowed in the model. An iterative solution algorithm for the model is proposed. Also, the model is extended to the case of variable vehicle capacity. Computational results verify the proposed model as well as the efficiency of the algorithm.

Keywords: Logistics management; Multi-item; Integrated models; Transportation costs

Introduction

Recently, a new approach to the analysis of production and distribution operations has been identified, which has proven to be of significant relevance to companies that have adopted it. This approach is based on the integration of decisions of different functions (e.g. supply process, distribution, inventory management, production planning, facilities location, etc.) into a single optimization model (Sarmiento and Nagi,1999). In this paper the integration of production, inventory and transportation arising in a supplier–retailer logistic system is considered. When products are delivered from the supplier to the consumer, transportation costs are incurred. In the traditional economic order quantity (EOQ) model, the transportation cost is calculated together with the production cost, or with the ordering cost. However, in a practical logistic system, the transportation cost of a vehicle includes both of the fixed cost and the variable cost. The fixed cost, which is considered to be a constant sum in each period, refers to some necessary expenses such as parking fare and rewards to the driver. As to the variable cost, it depends mainly on the oil consumed, which is related directly to the distance traveled. In this study, the problem of minimizing the production, inventory and transportation costs in multi-item case for a supplier–retailer logistic system is addressed. Both of the fixed cost and the variable cost of the vehicles are accounted in the model. In addition, since the multiple use of the vehicle can share the fixed cost and may reduce the total cost arising in the logistic system, the permitted working duration of the vehicle as well as the travel time of such vehicle along the trip are also considered. Other assumptions are as follows:

1. Shortage is not allowed.
2. Demand rate is deterministic
3. Capacities of vehicles are not identical.
4. Number of vehicle is given (or is decision variable, section 4-4)

5. Multistage shipment at each period is allowed.
6. The load of each vehicle may be less than or equal with the vehicle capacity.

In reviewing the literature of production-Inventory-Distribution-Inventory models (Mak and Wong,1995) propose the use of a genetic algorithm to solve the inventory-production-distribution problem. Their model consists of three echelons composed of several suppliers, one manufacturing plant and several retailers respectively. Their interest is to simultaneously obtain optimal stock levels, production quantities and transportation quantities so as to minimize the total system costs. These costs are inventory holding, shortage, manufacturing and transportation costs. (Zhao et al., 2004) present a model in a two-echelon system that optimizes the total cost including inventory cost in the second echelon and transportation cost in the first echelon. In their single item model, they considered both the fixed cost and the variable cost of the vehicles.

(Yano and Gerchak, 1989) present a solution methodology to simultaneously determine the safety stock level at the location in the second echelon (customer), number of vehicles required for regular delivery and time between shipments in such a way that overall operational costs are minimized. In the context of production-inventory-distribution-inventory models, (Blumenfeld et al., 1985a) are interested in analyzing the existing trade-offs between transportation, inventory holding and production set-up costs in the network. The authors analyze the cases of direct shipping between nodes in the echelons, shipping through a consolidation terminal and a combination of both, and obtain shipment sizes that trade-off the cited costs.

(Chien, 1993) determines the optimal production-shipping policies in one-to-one direct shipping with stochastic demand. An increasing amount of attention has been paid to the combined inventory and vehicle routing problems, which addresses the coordination of inventory and transportation management. A comprehensive review on this subject is provided by (Federgruen and Simchi-Levi, 1995). These problems are called inventory routing problems (IRP) and widely used in VMI partnerships. The study can also be viewed as an example of channel coordination problem (CCP), which has been studied by both marketing and supply chain researchers. (Huang and Lin,2010) present an integrated model that schedules multi-item replenishment with uncertain demand to determine delivery routes and truck loads. They utilized Ant colony algorithm for solving the model. (Liu and Chen,2011) proposed a mathematical model for the inventory routing and pricing problem (IRPP). They compared the result of the proposed heuristic method with that of two other methods in solving the model. In keeping with this trend, (Kutanoglu and Lohiya,2008) proposed an optimization model for an integrated inventory and transportation problem in a single-echelon, multi-facility service parts logistics system with time based service level constraints. They conclude that crucial advantages can be gained from transportation mode and inventory integration.

(Mendoza and Ventura, 2008) developed a traditional economic order quantity model with two modes of transportation, truckload (TL) and less than truckload (LTL) carriers. They used an exact algorithm to calculate optimal policies for single-stage models over an infinite planning horizon. (Bard and Nananukul, 2010) addressed a previously developed mixed-integer programming (MIP) model which minimizes production, inventory, and delivery costs across the various stages of the system. Their model consists of a single production facility, a set of customers with time varying demand, a finite planning horizon, and a fleet of homogeneous vehicles. They used branch-and-price framework to solve the underlying MIP.

The remainder of paper is organized as follows. Section 2 presents a new model for multi-item and some relevant lemmas. An algorithm used to find the optimal solution of the proposed model is given in Section 3. A few examples are followed in Section 4. Sensitivity analysis is carried out in Section 5, and finally conclusion is given in Section 6.

The model

The objective of this study is to minimize the whole average costs of the logistic system on the long planning horizon.

Notations:

W: Number of items

Kf: Fixed ordering cost for all items (per period)

Kfi: Fixed ordering cost of item i (per period)

Kv: Variable ordering cost for all item (per shipment)

Si: Purchasing (production) cost of item i

βi: The demand quantity per unit time (one day in this study) for item i

c: Variable cost per trip

f: Fixed transportation cost of type 1

v: Fixed transportation cost of type 2

p: The capacity of vehicle

t: Traveling time of each trip

r: Interest rate

m: Number of available vehicles

L: Lead time

T: Common order cycle time

n: Number of necessary trips for delivery of all demands.

m_i : The integer number of T intervals that the replenishment quantity of item i will lost.

In this study, we assume the vehicles are hired from the third logistic party whenever the delivery needs to be finished. The objective of the study is to minimize the whole average costs of the logistic system on the long planning horizon. Then the highest inventory occurs when $y=\sum_{i=1}^{w} y_i$ is received, and after T time periods the inventory quantity will be reduced to zero. Then the problem can be formulated as the following model (P_1):

MinTCU(T)=

$$\frac{1}{T}(k_f + \sum_{i=1}^{w}\frac{k_{fi}}{m_i}) + \frac{1}{T}(k_v\left\lceil\frac{n}{m}\right\rceil) + \frac{1}{T}\sum_{i=1}^{w} s_i\beta_i T + \frac{1}{T}nc + \frac{1}{T}mv + \frac{1}{T}mft\left\lceil\frac{n}{m}\right\rceil + \frac{1}{T}\left(H_A + H_B\right) \quad (1)$$

s.t.

$$n-1\frac{p}{\sum_{i=1}^{w} m_i\beta_i} < T \le n\frac{p}{\sum_{i=1}^{w} m_i\beta_i} \quad (2)$$

$$t\left\lceil\frac{n}{m}\right\rceil \le T \quad (3)$$

$$n, m, m_1, m_2, ..., m_w \in N \quad (4)$$

where TCU(T) is the total cost per unit time associated with the logistic system, y_i is the ordering size for item i, m is the number of vehicles used for delivering y, n is the total trips of these vehicles. The first and second term of objective function are ordering cost. The first term contains two cost components for each period. The first component relates to the ordering cost for all item; namely k_{f_i} .. The second component relates to the ordering cost for each item; namely k_{f_i} . The second term defines ordering cost for each stage of shipment. The third term defines the cost of material. The fourth term is variable transportation cost. The fifth and sixth term are fixed transportation cost. The fifth one relates to the hire of vehicle in the duration of shipment and sixth one is setup cost of vehicle in every period. The seventh term is inventory holding cost. To illustrate this latter term, suppose the length of period has been divided into two portions A and B as follows:

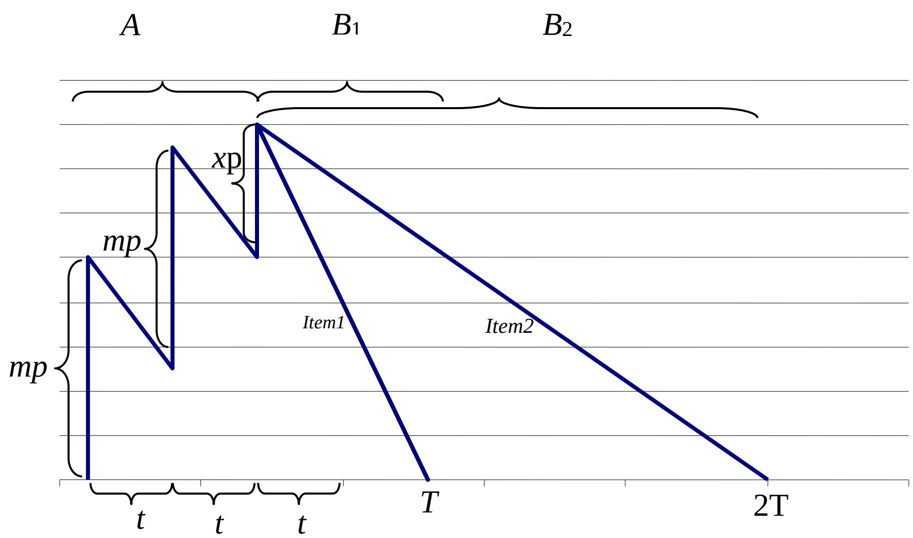

Figure 1: Inventory versus time

Portion A is portion of time that shipment of item is occurred and portion $B(=B_1+B_2)$ is portion of time that total inventory of item is consumed. Here it is assumed that we have two kinds of items. Item1 is ordered in every cycle and item2 is ordered once in every two cycles ($m_1=1$, $m_2=2$). B_1 is the duration that the inventory if item1 is consumed and B_2 is the duration that the inventory if item2 is consumed. The average inventory in the first section of portion A (namely A_1) is:

$$\bar{I}_{A_1} = \frac{mp + (mp - t\beta)}{2}$$

and similarly we have

$$\bar{I}_{A_j} = \frac{2jmp - (2j-1)t\beta}{2}$$

Where $\beta=\sum_{i=1}^{w} m_i\beta_i$. Hence, the inventory cost in portion A, namely H_A, would be as follows:

$$H_A = \sum_{j=1}^{\left\lfloor \frac{n}{m} \right\rfloor} \frac{2jmp - (2j-1)t\sum_{i=1}^{w} m_i\beta_i}{2} \times t \times \left(\sum_{i=1}^{w} \frac{rs_i}{w}\right) \qquad (5)$$

where $\sum_{i=1}^{w} rs_i / w$ is the average carrying charge. The average inventory in portion B during consummation of item2 is

$$\frac{\sum_{i=1}^{2} y_i - \left\lfloor \frac{n}{m} \right\rfloor t\beta_i}{2}$$

We have:

The duration of portion $B_1 = m_1 T - \left\lfloor \frac{n}{m} \right\rfloor t$

The duration of portion $B_2 = m_2 T - \left\lfloor \frac{n}{m} \right\rfloor t$

Hence, the inventory cost in portion B, namely H_B, is:

$$H_B = \sum_{i=1}^{w} (y_i - \left\lfloor \frac{n}{m} \right\rfloor t\beta_i)/2.(m_i T - \left\lfloor \frac{n}{m} \right\rfloor t).(\sum_{i=1}^{w} rs_i / w) = \sum_{i=1}^{w} \beta_i (m_i T - \left\lfloor \frac{n}{m} \right\rfloor t)^2 . \left(\sum_{i=1}^{w} rs_i / 2w \right) \quad (6)$$

Constraints (2) and (3) specifies the number of trips finished by the vehicles for delivering quantity y. If d is the number of allowable trips of each vehicle in the period, we will have $\frac{n}{m} \le d \le \frac{T}{t}$. Since d is an integer number, we have $\left\lceil \frac{n}{m} \right\rceil \le \frac{T}{t}$ and constraint (3) would be achieved.

$$\left. \begin{array}{l} (n-1)\dfrac{p}{\sum_{i=1}^{w} m_i \beta_i} < T \le n \dfrac{p}{\sum_{i=1}^{w} m_i \beta_i} \\ t\left\lceil \dfrac{n}{m} \right\rceil \le T \end{array} \right\} \Rightarrow \quad n\frac{p}{\sum_{i=1}^{w} m_i \beta_i} \ge t\left\lceil \frac{n}{m} \right\rceil \quad (7)$$

Since $t\left\lceil \frac{n}{m} \right\rceil \ge t\frac{n}{m}$, it is concluded that

$$n\frac{p}{\sum_{i=1}^{w} m_i \beta_i} \ge t\frac{n}{m} \Rightarrow \quad mp \ge \sum_{i=1}^{w} m_i \beta_i t \quad (8)$$

In the above relation, n is omitted. So, in the initial stage of problem solving, this relation can be considered as a feasibility check for problem. This relation means that the potential of shipment (mp) must meet, at least, the demand of duration of shipment ($\sum_{i=1}^{w} m_i \beta_i t$). TCU(T) is not a continuous function, it can not be differentiated during the whole interval. However, observe that when n is fixed and m_i,s are specified it can be differentiated. Denote TCU(T) with a given n and distinct m_i,s as $TCU_n(T)$, that is

$$TCU_n(T) = \frac{\gamma(n)}{T} + \frac{1}{T}\sum_{i=1}^{w} s_i \beta_i T + \frac{1}{T}\left[\frac{h}{2}\sum_{i=1}^{w} \beta_i (m_i T - \left\lfloor \frac{n}{m} \right\rfloor t)^2 \right] \quad (9)$$

Where

$$\gamma(n) = [k_f + \sum_{i=1}^{w} (\frac{k_{f_i}}{m_i}) + k_v \left\lceil \frac{n}{m} \right\rceil + nc + mft\left\lceil \frac{n}{m} \right\rceil + mv + H_A] \quad , \quad h = \sum_{i=1}^{w} rs_i / w$$

It is obvious that when y>0, function $TCU_n(T)$ is continuous. We obtain the derivation of relation (9) and set the result equal to zero; namely

$$\frac{dTCU_n(T)}{dT} = 0 \quad (10)$$

Then

$$T_n^{\ *} = \sqrt{\frac{2\gamma(n) + h\left\lfloor \frac{n}{m} \right\rfloor^2 t^2 \sum_{i=1}^{w} \beta_i}{h \sum_{i=1}^{w} m_i^{\ 2} \beta_i}} \tag{11}$$

and

$$TCU_n(T_n^{\ *}) = \sqrt{2h\left(\gamma(n) + \frac{h}{2}\left\lfloor \frac{n}{m} \right\rfloor^2 t^2 \sum_{i=1}^{w} \beta_i\right)\left(\sum_{i=1}^{w} m_i^{\ 2} \beta_i\right)} - h\left\lfloor \frac{n}{m} \right\rfloor t \sum_{i=1}^{w} m_i \beta_i + \sum_{i=1}^{w} s_i \beta_i \tag{12}$$

We wish to select the m_is to minimize $TCU_n(T_n^*)$. This is achieved by selecting the m_is

$$F(m_i's) = \sqrt{\left(\gamma(n) + \frac{h}{2}\left\lfloor \frac{n}{m} \right\rfloor^2 t^2 \sum_{i=1}^{w} \beta_i\right)\left(\sum_{i=1}^{w} m_i^{\ 2} \beta_i\right)} - h\left\lfloor \frac{n}{m} \right\rfloor t \sum_{i=1}^{w} m_i \beta_i$$

to minimize (13)

The minimization of Eq.13 is no simple matter because of two facts: (1) the m_is interact (that is, the effects of one m_i value depend on the values of the other m_is) and (2) the m_is must be integers (see (Schweitzer and silver,1983)). If we choose to ignore the integer constraints on the m_is and set partial derivatives of $F(m_i s)$ equal to zero (necessary conditions for a minimum), then

$$\frac{\partial F(m_i's)}{\partial m_j} = \frac{\left(\frac{-2hk_{f_j}}{m_j^{\ 2}} \sum_{i=1}^{w}\left(m_i^{\ 2}\beta_i\right)\right) + 2m_j\beta_j\left(2h\gamma(n) + h^2\left\lfloor \frac{n}{m} \right\rfloor^2 t^2 \sum_{i=1}^{w} \beta_i\right)}{2\sqrt{\left(2h\gamma(n) + h^2\left\lfloor \frac{n}{m} \right\rfloor^2 t^2 \sum_{i=1}^{w} \beta_i\right)\left(\sum_{i=1}^{w} m_i^{\ 2} \beta_i\right)}} - h\left\lfloor \frac{n}{m} \right\rfloor t\beta_j = 0 \tag{14}$$

Cross-multiplication and multiplication in m_j^2 gives

$$m_j^{\ 3}\,\frac{-\left\lfloor \frac{n}{m} \right\rfloor t\sqrt{\left(2h\gamma(n) + h^2\left\lfloor \frac{n}{m} \right\rfloor^2 t^2 \sum_{i=1}^{w} \beta_i\right)\left(\sum_{i=1}^{w} m_i^{\ 2} \beta_i\right)}}{\left(2\gamma(n) + h\left\lfloor \frac{n}{m} \right\rfloor^2 t^2 \sum_{i=1}^{w} \beta_i\right)} + m_j^{\ 2}\,\frac{-k_{f_j}\sum_{i=1}^{w} m_i^{\ 2}\beta_i}{\beta_j\left(2\gamma(n) + h\left\lfloor \frac{n}{m} \right\rfloor^2 t^2 \sum_{i=1}^{w} \beta_i\right)} = 0 \tag{15}$$

Suppose that there is a solution to these equations that results in

$$\left(-\frac{\left\lfloor \frac{n}{m} \right\rfloor t\sqrt{\left(2h\gamma(n) + h^2\left\lfloor \frac{n}{m} \right\rfloor^2 t^2 \sum_{i=1}^{w} \beta_i\right)\left(\sum_{i=1}^{w} m_i^{\ 2} \beta_i\right)}}{\left(2\gamma(n) + h\left\lfloor \frac{n}{m} \right\rfloor^2 t^2 \sum_{i=1}^{w} \beta_i\right)}\right) = a \tag{16}$$

$$\left(-\frac{k_{f_j}\sum_{i=1}^{w} m_i^{\ 2}\beta_i}{\beta_j\left(2\gamma(n) + h\left\lfloor \frac{n}{m} \right\rfloor^2 t^2 \sum_{i=1}^{w} \beta_i\right)}\right) = b \tag{17}$$

Then, from Eq.15, Eq.16 and Eq.17, we have

$$m_j^{\ 3} + am_j^{\ 2} + b = 0 \tag{18}$$

The solution of Eq.18 is

$$m_j = \sqrt[3]{-\left(\frac{b}{2}+\frac{a^3}{27}\right)+\sqrt{\frac{-a^8+a^6}{729}+\frac{a^3b}{27}+\frac{b^2}{4}}} + \sqrt[3]{-\left(\frac{b}{2}+\frac{a^3}{27}\right)-\sqrt{\frac{-a^8+a^6}{729}+\frac{a^3b}{27}+\frac{b^2}{4}}} - \frac{a}{3} \tag{19}$$

Model P_2 can be expressed as the following formulation:

$$\min_{n,m,m_1,m_2,\dots,m_w \in N} \begin{cases} \min TCU_n(T) \\ st(2,3) \end{cases}$$

Based on the above analysis and the m_is interact (that is, the effects of one m_i value depend on the values of the other m_is) and pay attention to this point that finding the best solution of the variables is no simple matter, the following conclusion and iterative solution procedure can be derived:

Conclusion. The function $TCU_n(T)$ is convex and there exists a unique lowest solution at the point $T_n = T_{n^*}$, where T_{n^*} can be given by relation (11).

Procedure. Use the following proposed iterative solution procedure to find the variables.

Set $m_1 = m_2 = ... = m_w = 1$.

Find the simultaneous solution of pair n and T.

Used the value of n determined in the previous step to find a new m_is.

If there are no changes in the new m_is values, then stop; otherwise go to step 2.

Because of the convex nature of the functions involved, convergence to the true simultaneous solution of pair (m_js and n) is ensured.

It is time consuming and impractical to compare object function for each $n \in N$ and $m_i s \in N$. However, based on the following theorems, we can limit the number of n need to be considered and the optimal solution of model P_2 can be found within limited steps.

Theorem 1. For any Function $TCU_n(T)$, if T_{n^*} gained by formulation (11) also satisfies constraints (2) and (3), then f(n)= $TCU_n(T_{n^*})$ (For different positive integer n, the lowest solution $TCU_n(T)$ which satisfies constraints (2) and (3) that denoted with value as f(n).); otherwise, f(n)=min{ $TCU_n(T_1)$, $TCU_n(T_2)$ }, where

$$T_1 = \frac{(n-1)p+1}{\sum_{i=1}^{w} m_i \beta_i},\ T_2 = n\frac{p}{\sum_{i=1}^{w} m_i \beta_i}.$$

Proof. Based on conclusion 1, we know that $TCU_n(T)$ is convex and there exists a unique optimal solution at $T_n = T_{n^*}$. If T_{n^*} is within the interval given by constraints (2) and (3), clearly f(n)= $TCU_n(T_{n^*})$. On the other hand, if T_{n^*} is not within the interval given by constraints (2) and (3), since the function $TCU_n(T)$ is either increased or decreased within the given interval, we can find the lowest value of $TCU_n(T)$ by comparing the value of the two side nodes of the interval.

Theorem 2. If $f(n_j)$ satisfies either of the following two conditions, then for all $n_i > n_j$, the lowest value of $TCU_{n_i}(T)$ that is, $f(n_i)$, cannot be lower than $f(n_j)$:

1. $f(n_j) = TCU_{n_j}(T_{n_j^*})$;
2. $f(n_j) \le TCU_{n_k}(T_{n_k^*})$, Where $n_k = n_j + 1$.

Proof.

1. As $\gamma(n)$ is a non-decreased function of n, it can be seen from formulations (11) and (12) that, with the increment of n, T_{n^*} and $TCU_n(T_{n^*})$ become larger. So, for all $n_i > n_j$, $TCU_{n_j}(T_{n_i^*}) > TCU_{n_j}(T_{n_j^*})$ then $f(n_i) \geq TCU_{n_i}(T_{n_i^*}) > TCU_{n_j}(T_{n_j^*}) = f(n_j)$.

2. It can be deduced from the given condition that $f(n_j) \leq TCU_{n_k}(T_{n_k^*}) \leq f(n_k)$. Since $TCU_{n_k}(T_{n_k^*})$ is a non-decreased function of n_k, we can further conclude that for all $n_i > n_k$, $f(n_j) \leq TCU_{n_k}(T_{n_k}^{*}) < TCU_{n_i}(T_{n_i}^{*}) \leq f(n_i)$.

Corollary. If $TCU_{\min}(n_j) \leq TCU_{n_k}(T_{n_k^*})$, where $TCU_{\min}(n_j) = \min\{f(n_r) \mid \text{for all } n_r \leq n_j\}$, then for all $n_i > n_k$, $TCU_{\min}(n_j) \leq TCU_{n_k}(T_{n_k^*}) < TCU_{n_i}(T_{n_i^*}) \leq f(n_i)$.

Proof. It can be deduced from above theorems.

The compound algorithm

Based on the above analysis, following algorithm that called the compound algorithm is presented for the model.

Step1. If $mp < \sum_{i=1}^{w} m_i \beta_i t$ then stop

Step 2. Set $m_1 = m_2 = ... = m_w = 1$.

Step 3. Let n=1.

Step 4. If there is not feasible solution for n then n=n+1.

Step 5. Calculate $T_{n^*} = \sqrt{\dfrac{2\gamma(n) + h\left\lfloor \dfrac{n}{m} \right\rfloor^2 t^2 \sum_{i=1}^{w} \beta_i}{h \sum_{i=1}^{w} m_i^2 \beta_i}}$.

Step 6. If T_{n^*} satisfies constraints $\dfrac{(n-1)p}{\sum_{i=1}^{w} m_i \beta_i} < T \leq n \dfrac{p}{\sum_{i=1}^{w} m_i \beta_i}$ and $t\left\lceil \dfrac{n}{m} \right\rceil \leq T$, go to step 7; otherwise go to step 9.

Step7.Record

$$f(n) = TCU_n(T_n^{*}) = \sqrt{2h\left(\gamma(n) + \frac{h}{2}\left\lfloor \frac{n}{m} \right\rfloor^2 t^2 \sum_{i=1}^{w} \beta_i\right)\left(\sum_{i=1}^{w} m_i^2 \beta_i\right)} - h\left\lfloor \frac{n}{m} \right\rfloor t \sum_{i=1}^{w} m_i \beta_i + \sum_{i=1}^{w} s_i \beta_i$$

$TCU_{\min} = \min\{TCU_{\min}, f(n)\}$, (the initial value of $TCU_{\min}$ is set to infinite).

Step 8. Use n determined from Eq.19 and obtains m_j. If there are no changes in m_j values, then stop; otherwise go to step 3.

Step 9. Calculate $T_1 = Max\left\{ \dfrac{(n-1)p+1}{\sum_{i=1}^{w} m_i \beta_i}, t\left\lceil \dfrac{n}{m} \right\rceil \right\}, T_2 = n \dfrac{p}{\sum_{i=1}^{w} m_i \beta_i}$.

$f(n) = \min\{TCU_n(T_1), TCU_n(T_2)\}$, $TCU_{\min} = \min\{TCU_{\min}, f(n)\}$, (the initial value of $TCU_{\min}$ is set to infinite).

Step 10. Calculate $TCU_{n_k}(T_{n_k^*})$ for $n_k = n+1$.

Step 11. If $TCU_{\min} > TCU_{nk}(T_{nk}^*)$ then n=n+1 and go to step 4.

Step 12. Use n determined from Eq.19 and obtains m_j. If there are no changes in m_j values, then stop; otherwise go to step 3.

The examples

Four examples are given in this section in order to verify the given model as well as the algorithm. In the first example, it is assumed that the transportation cost is proportional to the quantities delivered and no traveling duration constraint is considered. In the second example, the transportation cost is calculated based on travel distance of the vehicles and no fixed cost is considered. Whereas in the third example, the transportation costs include not only the fixed cost which is a fixed sum whenever a vehicle is employed, but also the variable cost which is calculated based on the travel distance of the vehicle. In the fourth example the capacity of the vehicles can be different. In other hand these four examples consider the whole aspects of the model. The meanings of the parameters in the examples are the same as those in the prior sections. In addition, for the purpose of simplicity, the units of the parameters in the examples are omitted. It is reasonable since the computational results as well as the conclusions can not be affected by such simplification.

Example 1

Suppose we have the following values for parameters.

$\beta_1 = 30, \beta_2 = 25, \beta_3 = 45, k_f = 38.5, k_{f_1} = 10.5, k_{f_2} = 7, k_{f_3} = 14, k_v = 0, r = 0.077, s_1 = 0.25, s_2 = 0.2,$ s_3=0.30,p=25,c=f=v=0,t=0.5 In addition, we assume the transportation cost is proportional to the quantity delivered and the unit transportation cost, defining by a, equal to 0.2. The objective is to decide optimal value of T^* and y_{i^*} (i=1,2,3) with the respect of minimizing the total average cost of the logistic system, where T^* is the ordering cycle, y_{i^*} is the economic order quantity for item i. We use the following simple procedure (Silver and Peterson, 1998) to find the best set of m_i,s and y_{i^*},s .

Step1. Evaluate $\frac{k_{fi}}{s_i \beta_i}$ for all items such that $\frac{k_{fi}}{s_i \beta_i}$ is smallest for item i. Set $m_i = 1$.

Step2. For $j \neq i$ evaluate $m_j = \sqrt{\frac{k_{fj}}{s_j \beta_j} \frac{s_i \beta_i}{k_{fi} + k_f}}$ rounded to the nearest integer greater than zero.

Step3. Evaluate T^* using the $$T^* = \sqrt{\frac{2(k_f + \sum_{i=1}^{w} \frac{k_{f_i}}{m_i})}{r \sum_{i=1}^{w} s_i m_i \beta_i}}$$.

Step4. Determine $y_{i^*} = m_i \beta_i T^*$.

$\frac{k_{f_1}}{s_1\beta_1} = 1.4, \frac{k_{f_2}}{s_2\beta_2} = 1.4, \frac{k_{f_3}}{s_3\beta_3} = 1.03 \Rightarrow m_3 = 1$ and $m_1 = \sqrt{\frac{k_{f_1}}{s_1\beta_1} \frac{s_3\beta_3}{k_3 + k}} = 0.36 \Rightarrow m_1 = 1$ and $m_2 = \sqrt{\frac{k_{f_2}}{s_2\beta_2} \frac{s_3\beta_3}{k_3 + k}} = 0.36 \Rightarrow m_2 = 1$

Thus in each ordering cycle, items 1, 2, 3 are ordered.

$$T^* = \sqrt{\frac{2(k_f + \sum_{i=1}^{w} \frac{k_{f_i}}{m_i})}{r\sum_{i=1}^{w} s_i m_i \beta_i}} = 8.36$$

, $y_1^* = 251, y_2^* = 209, y_3^* = 376$ and $y^* = \sum_{i=1}^{3} y_i^* = 836$

$$TCU(T^*) = \sqrt{2r(\sum_{i=1}^{3} s_i m_i \beta_i)(k + \sum_{i=1}^{3} \frac{k_i}{m_i})} + \sum_{i=1}^{3} (s_i + a)\beta_i = 16.72 + 46 = 62.72$$

The results show that in the optimal solution, an order for 836 unit products consist of 201 unit item 1, 209 unit item 2, 376 unit item 3, should be sent. The optimal ordering cycle is 8.36 days and the total transportation cost in each ordering cycle is 167.2. Base on the results, the next example is given.

Example 2

It is assumed that,

$\beta_1 = 30, \beta_2 = 25, \beta_3 = 45, k_f = 38.5, k_{f_1} = 10.5, k_{f_2} = 7, k_{f_3} = 14, k_v = 15, r = 0.077, s_1 = 0.25, s_2 = 0.2,$ $s_3 = 0.30, p = 200, c = 40, f = v = 0, t = 0.5, m = 3$. We would like to find the optimal solutions of T^* and y_i^* (i =1,2,3). The algorithm described in Section 4 is coded by Matlab. The computational results are showed in Table 1.

Table 1: Computational results of Example 2

n	T_1	T_2	T_n^*	$TCU_n(T_n^*)$	$TCU_n(T_1^*)$	$TCU_n(T_2^*)$	f(n)	TCU_{min}
1	0.5	2.00	11.40	47.92	276.48	90.42	90.42	90.42
2	2.01	4.00	13.10	51.19	110.02	71.09	71.09	71.09
3	4.01	6.00	14.81	53.51	81.45	65.93	65.93	65.93
4	6.01	8.00	16.63	57.01	75.04	65.95	65.95	65.93
5	8.01	10.00	17.83	59.33	70.91	65.23	65.23	65.23
6	10.01	12.00	19.28	61.14	69.39	65.39	65.39	65.23
7	12.01	14.00	20.71	63.89	69.95	66.98	66.98	65.23
8	14.01	16.00	21.69	65.78	-	-	-	-

It can be seen from the results that, based on the stopping criterion in step 12, the algorithm stops when n=8. The optimal solution occurs at n=5, and $T^* = 10$, $y_1^* = 300$, $y_2^* = 250$, $y_3^* = 450$, $y^* = 1000$, $TCU_{min} = 65.23$

The reason that the results of the above two examples are equal is that we design example 2 based on the results from example 1, that is, the parameters given in example 2 guarantee that vehicle used for delivery of y^* in example 1 is fully loaded. However, in example 2, when n = 5, the value of T_n^* calculated according to relation (11) is 17.82, whereas based on the algorithm, T^* equals to 10. In other words, the solution method using the traditional economic ordering quantity formula is not suitable for the given problem in this example.

We can also see from the results that for all $n \le 8$, $f(n) > TCU_n(T_{n^*})$, and the value of $TCU_n(T_{n^*})$ increase along with the increment of n. Following computation indicates that when n = 15, $f(n) = TCU_n(T_{n^*}) = 77.74 > TCU_{min}$. Such results further verify the algorithm.

Example 3

It is assumed that,

$\beta_1 = 30, \beta_2 = 25, \beta_3 = 45, k_f = 38.5, k_{f_1} = 10.5, k_{f_2} = 7, k_{f_3} = 14, k_v = 15, r = 0.077, s_1 = 0.25, s_2 = 0.2$
$s_3 = 0.30, p = 200, c = 40, f = 30, v = 0, t = 0.5, m = 3$.

The computational results are presented in Table 2.

Table 2: Computational results of Example 3

n	T_1	T_2	T_n^*	$TCU_n(T_n^*)$	$TCU_n(T_1^*)$	$TCU_n(T_2^*)$	F(n)	$TCU_{\min}$
1	0.50	2.00	13.30	51.67	366.48	112.92	112.92	112.92
2	2.01	4.00	14.78	54.41	132.41	82.34	82.34	82.34
3	4.01	6.00	16.31	56.40	92.67	73.43	73.43	73.43
4	6.01	8.00	19.24	62.02	90.01	77.20	77.20	73.43
5	8.01	10.00	20.29	64.05	82.15	74.23	74.23	73.43
6	10.01	12.00	21.57	65.55	78.38	72.88	72.88	72.88
7	12.01	14.00	23.86	69.95	81.19	76.63	76.63	72.88
8	14.01	16.00	24.72	71.60	79.46	76.16	76.16	72.88
9	16.01	18.00	25.89	72.90	-	-	-	-

It can be seen from the results that, based on the criterion in step 12, the algorithm stops when n=9, The optimal solution occurs at n=6, and $T^* = 12$, $y_{1^*} = 360$, $y_{2^*} = 300$, $y_{3^*} = 540$, $y^* = 1200$, $TCU_{\min} = 72.88$

If n = 17, then $f(n) = TCU_n(T_{n^*}) = 89.08 > TCU_{\min}$.

When the fixed cost of the vehicle is considered, the value of T^* and y^* are different from those gained in example 2. It shows that under n = 5, one of the vehicles used will finish only one trip, thus the marginal transportation cost of such vehicle for delivering unit product will be higher contrasting to that of other vehicles which can complete two trips. On the other hand, if the vehicles are utilized to the greatest efficiency, the inventory quantities may increase. So the optimal solution of the problem is the results of the trade-off of the transportation cost and the inventory cost. We are interested to know, although the model has been developed based on constant m, which m achieves the minimum cost. In fact, we look for the optimal vector of (T^*, n^*, m^*). Since m is limited and can not adopt usual values, selecting different values for m, from possible smallest value to possible longest value, and substituting them in the model and running the algorithm, the least TCU^* and corresponding m^* will be determined easily.

In fact, by the integration of this small algorithm and algorithm of model, a new algorithm (we call it compound algorithm) is resulted through which the model can be solved optimally with three decision variables. With respected to the feasibility condition of problem and first setup of algorithm and since m is an integer number, the possible smallest value for m is:

$$m = \left\lfloor \frac{t\sum_{i=1}^{w} m_i \beta_i}{p} \right\rfloor + 1 \tag{20}$$

Suppose at most 100 vehicles are available for the company, and the parameters of the model are as follows:

$\beta_1 = 30, \beta_2 = 25, \beta_3 = 45, k_f = 38.5, k_{f_1} = 10.5, k_{f_2} = 7, k_{f_3} = 14, k_v = 30, r = 0.077, s_1 = 0.25, s_2 = 0.2$
$s_3 = 0.30, p = 25, c = 40, f = 30, v = 3, t = 0.5$

Algorithm has been run for m=5 to m=100, and the optimal value of cost versus m has been depicted in figure 2.

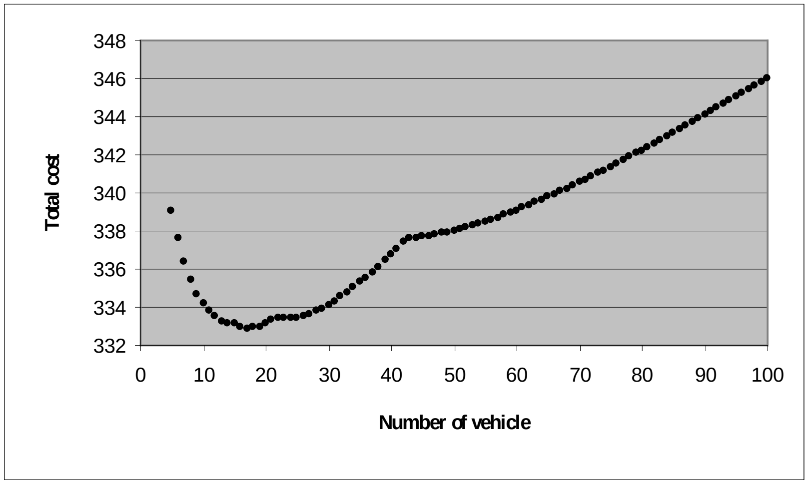

Figure 2: The total cost resulted from the algorithm for different m_i,s

As it is observed, the curve of Figure 2 demonstrates complexity and even existence of local optimal points. The least cost occurs in m=17 with the value of 332.88. Algorithm stopped after 409 iterations. Table 3 gives the solution of problem.

Table 3: Output of algorithm for m=17

Results	Costs
n=51	Ordering=12.55
m=17	Purchase=26
T^* =12.75	Variable shipment=160
y_1^* =382.5	Constant shipment 1=120
y_2^* =318.75	Constant shipment 2=4
y_3^* =573.75	Inventory=10.33
y^* =1275	Total=332.88

Example 4

We discussed the problem further by considering the combination of vehicle through the presentation of an example. Suppose vehicles with different capacities are available and we want to select an optimal combination of them to ship the economic order quantity of items. We run the model for different capacities and obtain the total cost for each special capacity. Then we find the solution of problem through running a knapsack model. But in implementing the model, what input data for m should be given to the model?

We can run the model for each special capacity for different m_i,s and find the optimal m (application of compound algorithm). The total cost achieved by optimal m is utilized for knapsack model. Suppose that the parameters are as given in Table 4 and five type of vehicles with different capacity are available:

$$\beta_1 = 60, \beta_2 = 50, \beta_3 = 90, k_f = 38.5, k_{f_1} = 10.5, k_{f_2} = 7, k_{f_3} = 14, k_v = 30, r = 0.077, s_1 = 0.25,$$

$$s_2 = 0.2, s_3 = 0.3, t = 1$$

For each capacity and corresponding parameters, the model has been repeatedly implemented and m^* has been brought in Table 4 for each capacity.

Table 4: Parameters of Example 4

P	v	c	f	m^*	TCU^*
50	3	30	25	12	307.23
75	4	35	28	8	254.79
100	5	95	85	6	446.57
125	6	16	15	5	598.44
150	7	19	18	4	593.01

Now the knapsack model is as follows:

$$\text{Min}\, 307.22X_1 + 254.79X_2 + 446.57X_3 + 598.44X_4 + 593.01X_5 \tag{21}$$

s.t.

$$50X_1 + 75X_2 + 100X_3 + 125X_4 + 150X_5 = 200 \tag{22}$$

$$X_i \geq 0, \text{ Integer} \qquad i = 1,2,\ldots,5 \tag{23}$$

After the model is solved, we have: $X_1 = 1, X_2 = 2, X_3 = 0, X_4 = 0, X_5 = 0$

So, we should use vehicle with the capacity 50 at the number of 12 ($X_1 \times m_1^*$) and vehicle with the capacity 75 at the number of 16 ($X_2 \times m_2^*$)

Sensitivity analysis

The logical sensitivity of model towards a parameter indicates the validity of model. Sensitivity analysis has been performed with respect to the change in the value of parameters, and performed for example 3. The results have been shown in figures 3 to 8. The number of the vehicles has been considered unconstrained to determine its optimal value.

In the considered example, the fixed ordering cost $(k_f + k_{f_1} + k_{f_2} + \ldots + k_{f_w})$ is taken 70 units and variable ordering cost (k_v) is taken 30 units. Now, suppose that the sum of values of these two parameters remains constant and the ratio of parameters is changeable. If variable ordering cost contributes a large portion of total ordering cost , to avoid this expensive cost, logically the number of travels for transportation in one period should be reduced. In order words, n^* should be inclined to m^*. In the opposite direction, in the case that fixed ordering cost contributes a large portion of total ordering cost, the issue of ordering should be done in such a way that much more number of travels is managed within one ordering. Meanwhile, if ordering costs can permit to increase the number of travels, then the possibility of reduction of number of vehicles exists to save other costs like hire of vehicles (Figure3).

With constant m (number of vehicles), certainly with increase in consumption rate, higher volume of items should be shipped through increase in the number of travels (Figure3). In the corresponding curve, one optimal point, resulting from differentiating, exists.

Suppose when m is constant, suddenly the capacity of vehicles increases. Definitely, in the case, the number of travels will be reduced. In other word, n^* will decrease. From other side, if number of travels (n^*) is constant, with increase in the capacity, higher volume of items will be shipped; namely, y^* (and T^*) will increase. If the number of vehicles is determined optimally, obviously, with increase in the capacity, lesser number of vehicles is required. In addition the point is that, p is an important factor in the first constraint; the sensitivity of model towards this parameter will increase (Figure 5).

If interest rate increases, the logical consequence of the event will result in reduction in inventory and repetitive ordering. In the case of variable vehicle (m), using compound algorithm, as we see the ordering size gradually reduces in the direction of inventory reduction, also the number of vehicles (m^*) reduces gradually with the reduction in the shipment requisitions (Figure 6).

If the fixed transportation cost of second type (setup) increases, the logical consequence is that attempts are made to increase the length of period; so that higher volume of items is shipped with one time payment of setup cost. Increase in the length of period is direct relation with n^*. In addition, the next consequence is reduction in the number of vehicles (m^*). It is true because setup cost belongs to each vehicle in one period and, to reduce this cost, no way except decrease in the number of vehicles exists (Figure 7).

Travel time (t), like the capacity of vehicles (p), is an important parameter that model shows a high sensitivity towards it. This parameter is present in the second constraint; in addition of its existence in some terms of cost function. In the case of constant number of vehicles, with increase in the time of travel (t), n^* and T^* will increase gradually. With increase in t, in the case that optimality of n^* and T^* dose not change, the inventory cost will decrease. The model intends to issue higher quantity of order to reduce fixed ordering cost and vehicle setup cost. Hence, increase in n^* and T^* are observed in graphs (Figure 8).

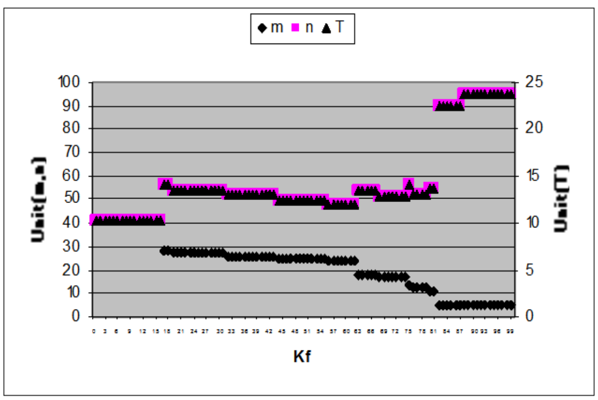

Figure 3: Sensitivity analysis of ordering cost

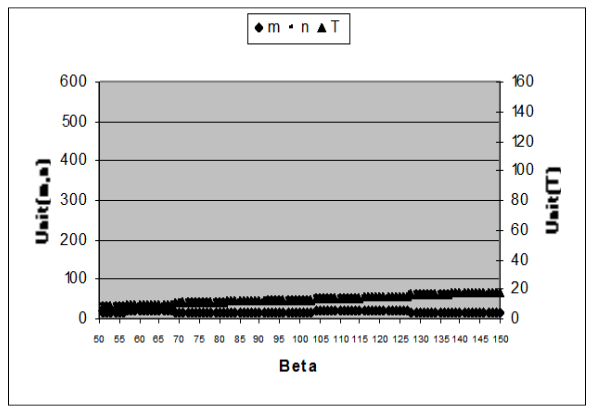

Figure 4: Sensitivity analysis of consumption rate of items

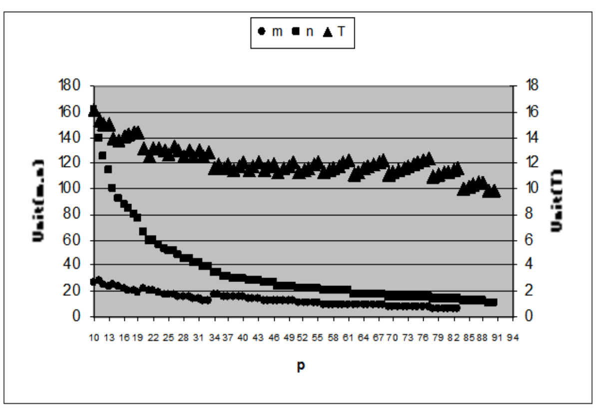

Figure 5: Sensitivity analysis of transportation capacity of vehicles

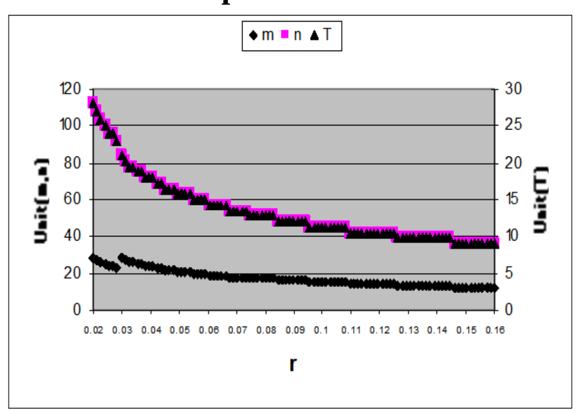

Figure 6: Sensitivity analysis of interest rate

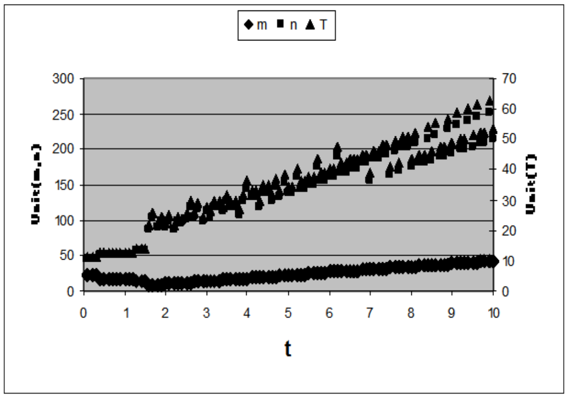

Figure 7: Sensitivity analysis of fixed transportation cost of second type (setup)

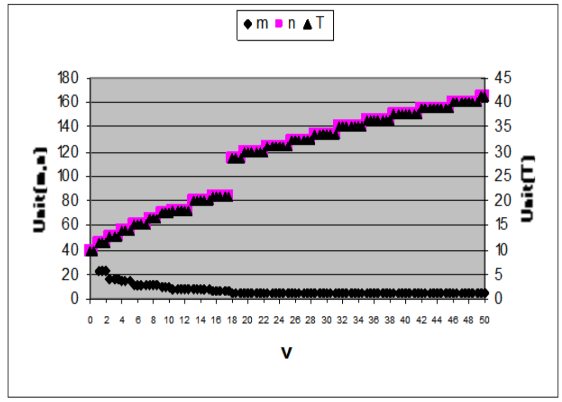

Figure 8: Sensitivity analysis of travel time

Conclusions

In this paper a model was developed to study the behavior of a logistic system of supplier retailer with multiple items. The considered costs are ordering cost, inventory cost, material cost and transportation cost. This model concerns decision making in both tactical and strategic levels. In tactical level, the operational decisions are made like optimal order cycle (and subsequently optimal order size) and the mode of transportation. In strategic level, the decision are developed model was nonlinear with several decision variables, through presentation of some theorems, a solution algorithm was proposed to give optimal solution of problem. This algorithm was finally implemented and an example was given.

In sensitivity analysis, the sensitivity of model towards each of parameters was evaluated. To show efficiency, the algorithm was implemented for tens of thousands of times and the logical behavior of model was analyzed and validated. This behavior indicates the validity and applicability of model. Model supposes that supplier and retailer (consumer) are unique in every step. Consideration of multiple suppliers or multiple retailers causes the model to be more comprehensive. Meanwhile, we can add other decision variables for other application like inventory management, distribution and production planning to extend the context of system integrity, Also, the allocation of different types of vehicles to items can be recommended as another dimension of future development of model. It means that the special items should be transported with vehicles of special capacity. Classification of transportation based on some measures of priorities can be respected in this area of research. Finally, the assumption of shortage is another area recommended for future studies.

References

Bard, JF, Nananukul, N (2010). A branch-and-price algorithm for an integrated production and inventory routing problem. J. Comp. Oper. Rsrch.,37: 2202–2217

Blumenfeld, D. E., Bums,L. D., Diltz, J. D., Daganzo, C. F.; (1985a). Analyzing trade-offs between transportation, inventory and production costs on freight networks, Transportation Research 19B(5), 361-380.

Chien, T. W.; (1993). Determining profit-maximizing production/shipping policies in a one-to-one direct shipping, stochastic demand environment, European Journal of Operational Research, 64, 83-102.

Federgruen, A., Simchi-Levi, D.; (1995). Analysis of Vehicle Routing and Inventory Routing Problems; Handbooks in Operations Research and Management Science, 8, 297-373. Network Routing, North-Holland, Amsterdam.

Huang, SH, Lin, PC (2010).A modified ant colony optimization algorithm for multi-item inventory routing problems with demand uncertainty. J. Trans. Rsrch. Part E.,46: 598–611

Kutanoglu, E, Lohiya, D (2008). Integrated inventory and transportation mode selection: A service parts logistics system. Trans. Rsrch.Part E: Logis. Trans. Rev., 44: 665–683

Liu, SC, Chen, JR (2011).A heuristic method for the inventory routing and pricing problem in a supply chain. J. Expert. Sys. App., 38: 1447-1456.

Mak, k. L., Wong, Y. S.; (1995); “Design of integrated production-inventory- distribution systems using genetic algorithm”; In Proceedings of the lst IEE/IEEE International Conference on Genetic Algorithms in Engineering Systems: Innovations and Applications, Sheffield, Engl., September 12-14, pp. 454-460. Sponsored by IEE.

Mendoza, A, Ventura, JA (2008). Incorporating quantity discounts to the EOQ model with transportation costs. Inter. J. Prod .Econ.,113, 754–765

Sarmiento, A.M, Nagi R (1999). A Review of Integrated Analysis of Production-Distribution Systems. IIE Transaction. 31: 1061-1074

Schweitzer, PJ, Silver, EA (1983).Mathematical Pitfalls in the One Machine Multiproduct Economic Lot Scheduling Problem. Oper.Rsrch., 31(2), 401-405

Silver, E. A., Pyke, D. F., Peterson, R.; (1998). Inventory management and production planning and scheduling; third edition; John Willey & Sons Inc.

Yano, C. A., Gerchak, Y.; (1989) Transportation contracts and safety stocks for Just-in-time deliveries, Journal of Manufacturing and Operation Management 2, 314-330.

Zhao, Q. H., Wang, S. Y., Lai, K. K., Xia, G. P.; (2004) Model and algorithm of an inventory problem with the consideration of transportation cost, Computer & Industrial Enginnering, 46, 389-397.