

## Modeling a DFIG-Based Wind Turbine Focusing on DFIG and Aerodynamic Models

Rabie Gohar<sup>1\*</sup>, F. Servati<sup>2</sup>

<sup>1</sup>Bharati Vidyapeeth University, College of Engineering, Pune, India; <sup>2</sup> Istanbul technical university, Department of Mechanical Engineering, Istanbul, Turkey

\*E-mail: Arash.rabie@yahoo.com

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### Abstract

The present study is an attempt to develop and implement mathematical models of variable speed wind turbines using Doubly-Fed Induction Generator (DFIG) including aerodynamic model and DFIG through newly developed Matlab/Simulink. The so called models are developed in order to investigate dynamic behavior of power networks as well as to design and evaluate the behavior of each wind turbine under normal and disturbance conditions. In the study, aerodynamic (rotor) model is based on torque coefficient  $C_q$  look up table which is according to BEM method. DFIG model is developed in two sub models including generator and power converters. In generator model, electrical dynamic equations of wound rotor induction generator are provided in two different models (fifth-order model and third-order model) by choosing synchronous rotating reference frame. As the simulation result revealed, the stator power is limited at rated value at high wind speed but wind turbine is operated at optimum power efficiency at lower wind speed to gain optimal power output, indicating a good agreement with the real behavior of the system.

**Keywords:** variable speed, wind turbine, Doubly-Fed Induction Generator

### Introduction

Wind energy is considered as one of the best technologies to provide a sustainable supply to the world development due to its abundant, inexhaustible potential, its increasingly competitive cost, and environmental advantage (Erlich et al, 2007). During the recent years, with increasing development of wind energy conversion technology, total installed capacity of the wind turbines has reached as much as 39.234GW by the end of 2004 and will exceed 110GW by 2012. Today, the wind turbines available in the market adhere to innovative concepts in technology and pertain to either fixed speed or variable speed applications (Ackerman, 2005).

In variable speed wind turbine, the power efficiency coefficient of the system reaches its maximum value leading to improved power output by continuously adjusting the rotor speed to wind speed (tip speed ratio) at optimal point and also at higher wind speed by controlling the pitch angle of turbine blades. In this sense, wind turbine operation in wide range of wind speed variation (from limited-variable to full-variable speed) has become possible through power frequency converters compensating the difference between variable mechanical frequency and fixed electrical frequency by injecting a generator rotor current with a variable frequency according to the shaft speed. Variable-speed operation of wind turbines are used for some reasons including the possibility to reduce stresses due to the mechanical structure, acoustic noise reduction, power quality and system efficiency improvements, and the possibility to control active and reactive power.

During the recent years and with the development of generators and elements design such as power electronic devices, large variable speed wind turbines (i.e. in the range of 3 to 5 MW) equipped with Doubly-Fed Induction Generator (DFIG) have become as one of the most popular configurations with high penetration level for electricity generation due to their meritorious characteristics. Such scheme shows that Wound Rotor Induction Machine is connected to power grid at two points, directly by stator and via rotor side through two identical self commutated Pulse-Width Modulated (PWM) circuits Voltage Source Converters (VSCs), (since voltage source converters have less tolerance to high current compared with conventional converter in case of large disturbances) which are connected back-to-back through DC-link and controlled independently to enable DFIG to transport power in both directions. The most important advantage of this concept is that the power electronic equipment only has to handle a fraction (20–30%) of the total system power. That is, the losses in the power electronic equipment can be reduced compared with power electronic equipment which has to handle the total system power as for a direct-driven synchronous generator (without gearbox) in full variable speed wind turbine, apart from the cost saving of using a smaller converter (Peterson, 2005).

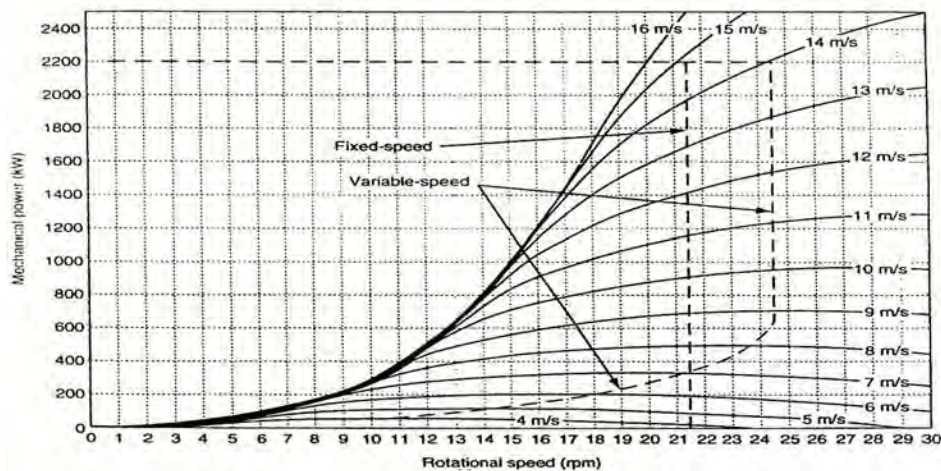
By using DFIG, the energy capture can be significantly increased by over 20% with respect to a variable-speed system using a cage-bar induction machine and by over 60% in comparison to a fixed-speed system. Nevertheless, according to the considered assumptions, these values are not fixed and can vary from investigation to investigation. Factors such as speed control of variable-speed WTs, blade design, the kind of power needed for comparison, selection of maximum speed of WT, selected blade profile, missing facts regarding the base assumptions, and etc. influence investigations outcomes (Peterson, 2005).

The present study focuses on developing and implementing mathematical model of variable-speed WTs using new Matlab/Simulink toolbox in order to investigate dynamic interaction between large wind turbines/farms and power networks as well as design and evaluate wind turbine behavior under normal and disturbance conditions.

These models are developed due to the high growing penetration level of this scheme into power networks (from 26.5% in 1998 to 40.8% in 2002) and to ensure appropriate transient and small signal stability of the grid. The developed models compromise between accuracy and simplicity required for large systems simulation, enabling both the potential wind turbine owners and the grid utility technical staff to conduct the necessary studies before investing and connecting wind turbines/farms to the grid.

### **Variable Speed Wind Turbines**

During the last years, variable speed wind turbines have been widely used among the installed wind turbines (Ackerman, 2005) since the variable speed wind turbines are designed to gain maximum aerodynamic efficiency and thereby to extract maximum power output over a wide range of wind speed variation compared to fixed speed wind turbines designed to obtain maximum power efficiency at one wind speed only. Figure 1 illustrates that the mechanical power output of variable speed wind turbine is higher than similar fixed speed wind turbines at all wind speeds except at wind speed of 7m/s. At this point, the mechanical power output of both types is the same (Ackerman, 2005).



**Figure 1- Mechanical power curves of fixed speed and variable speed wind turbine**

The purpose of variable speed wind turbine at lower speed (i.e. wind speed above the cut-in speed and below the turbine rated speed) is to adjust the rotor speed at changing wind speeds so that  $C_p(\lambda, \beta)$  always is maintained at its maximum value for an optimum pitch angle and high speed ratio; that is, the control strategy has to change the turbine rotor speed in such a way that the optimum tip speed ratio  $\lambda_{opt}$  is always obtained to extract maximum power from particular wind speed (Hansen et al, 2007). At wind speed higher than rated wind speed, the control has to be changed so that the wind turbine no longer produce maximum power but only rated power and it can be achieved by turning the blade away from the optimal pitch angle. It can be done either out of the wind or up against the wind (Ackerman, 2005). If the blades are turned out of the wind, the lift on the blades is gradually reduced by the decrease of the attack angle  $\alpha$ . It is called active pitch control needing a relatively large change in pitch angle to make a significant reduction in power. When the blades are turned up against the wind, the airfoil attack angle  $\alpha$  is increased towards stall and subsequently, the lift on the turbine blade is reduced. Such effect is obtained by a relatively small change in pitch angle. It is called stall control requiring more accurate control of the pitch angle since the high angular sensitivity variable speed active stall controlled wind turbines are not included here as potentially they lack the capacity for a fast reduction of power. In case of running wind turbine at maximum speed and availability of strong gust, aerodynamic torque can get critically high and may lead to a runaway situation.

In fact, pitch control allows the power input of generator to be cut off at exactly the desired level, leading to a flat power curve (i.e. the power output in kw as a function of wind speed) all the way from the rated wind speed out to the cut-out speed when the machine is shut down to be protected.

From electrical point of view, operation of variable speed wind turbines in wide range of wind speed from limited-variable to full-variable speed has become possible through power electronic devices providing a decoupling mechanism between fixed frequency of electrical system and variable speed mechanical system, as well as allowing to rotor speed to vary. It also allows controlling the power flow from AC generator (mostly induction generator) to the grid through which controls the air gap torque between the generator stator and rotor. The turbine speed is maneuvered to increase efficiency by controlling the air-gap torque precisely according to a schedule defining sensed generator speed versus sensed generator power output (Doman et al, 1987). The advantages of variable-speed wind turbines which can be stated as follow:

1. Improving system efficiency
  2. Reducing mechanical stress on gear box and drive train
  3. Reducing acoustic noise
  4. Keeping small the gearbox torque variations above mean rated level by direct controlling of the air-gap torque between the generator stator and rotor
  5. Improving power quality
  6. Controlling both active and reactive power (frequency and voltage stability) to maintain unity power factor
  7. Compensating poor power factor of other consumers on the network leading to a positive influence of variable speed wind farm in network stability
  8. Offering island operation capability (Ekanayake et al, 2003)
- However, significant cost associated with the power converter devices, increased losses, increased complexity due to the use of more components and controllers, generation of electrical noise and high harmonic current by the inverter system can be referred as the drawbacks of variable speed wind turbines (Burton et al, 2001).

### **Classification of Variable Speed Wind Turbines**

There are two types of variable-speed wind turbines in the market (Burton et al, 2001):

1. Full variable-speed concept in which generator can be synchronous (WRSG, PMSG) or induction generator (WRIG) connected to grid through the full-scale frequency converter. This type of WT can be connected to the generator without speed increasing gearbox; hence, it is now as direct drive wind turbine.
2. Limited variable speed concept in which the generator stator is connected to grid so that the rotor frequency and speed is controlled. The generator is wound rotor induction generator (WRIG). This type is classified in two classes of variable generator rotor resistance (variable-slip) and doubly fed induction generator (DFIG).

Figure 2 depicts the configuration of a typical variable speed WT.

### **Doubly-Fed Induction Generator**

During the recent years, with development of power electronic devices, large variable wind turbine equipped with doubly fed induction generator (DFIG) are considered as the most effective and popular configuration for electricity generation due to its advantages. The typical configuration of variable speed wind turbine with doubly-fed induction generator (DFIG) concept has been shown in Figure 3. As shown in the figure, the power captured by turbine from wind is converted into electrical power by a wound rotor induction generator as a kind of induction generator running at super synchronous speed and then, it is transmitted to the grid via two way (so-called doubly fed); from the stator windings directly connected to the three-phase grid and also with the rotor windings connected to a back-to-back (AC/DC/AC) partial scale power frequency converter via slip rings and brushless to allow current into or out of the rotating secondary windings. It consists of two independent controlled voltage source converters (since voltage source converters are less tolerant to high current than conventional converter in case of large disturbances) connected to a common DC-link capacitor; these converters are known as rotor side converter  $C_{rotor}$  and grid-side converters  $C_{grid}$  that use force-commutated power electronic devices (IGBTs) to synthesize an AC voltage from a DC voltage source (Ganon et al, 2005). A coupling inductor,  $L_f$ , acts as filter grid and is used to connect  $C_{grid}$  to the grid in order to obtain sinusoidal line currents. Further, a grid filter is placed between the  $C_{grid}$  and the grid since both the grid and the

voltage source converter are voltage stiff and also to reduce the high frequency content of the line current harmonics caused by the switched operation of the grid side converter. The control system generates the pitch angle command and the voltage command signal  $V_r$  and  $V_{gc}$  for  $C_{rotor}$  and  $C_{grid}$ , respectively, in order to control the power of the wind turbine, the DC bus voltage and the voltage at the grid terminal.

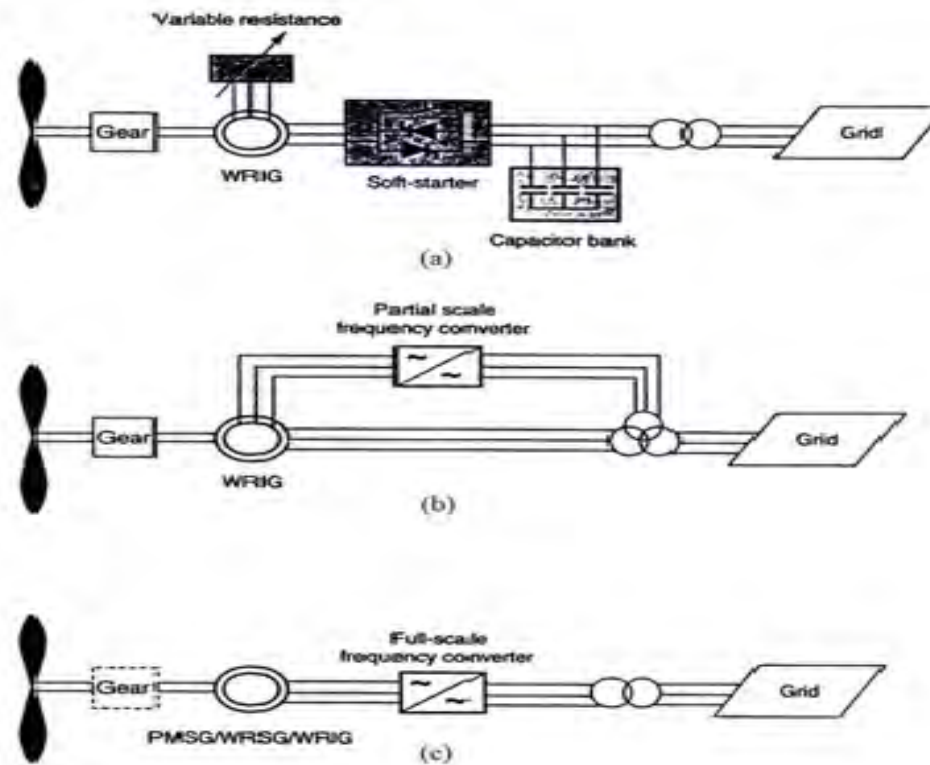


Figure 2- Typical variable speed wind turbine configurations

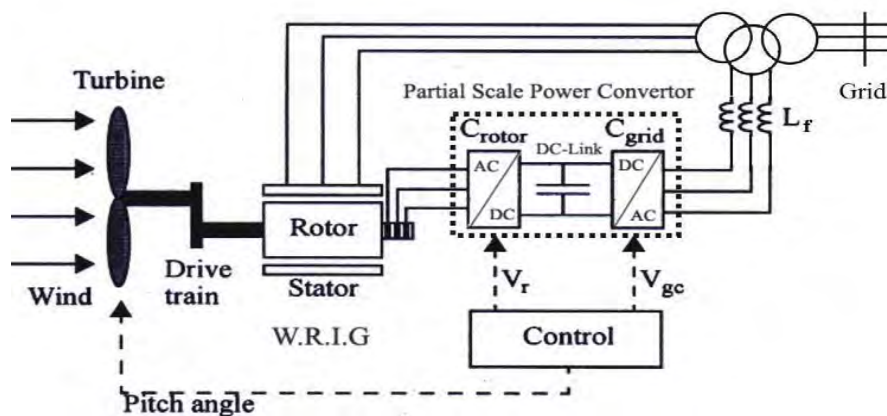


Figure 3-Variable speed wind turbine with DFIG concept



In DFIG, variable speed operation over a large but restricted range has become possible due to the presence of power converters that compensating the difference between the variable mechanical frequency and fixed electrical frequency by injecting a rotor current with a variable frequency according to the shaft speed. Through collector rings, the power converter supplied thus rotor windings with a voltage with variable magnitude and frequency. Hence, the behavior of generator is governed by the power converter and its controllers during both normal and fault condition (Ackerman, 2005). In other words, to keep constant frequency  $\omega_1$  of stator voltage of DFIG, rotor frequency  $\omega_2$  of AC excited generator fed by the bi-directional power converter should be adjusted according to following equation to accommodate varying rotational speed of wind turbine.

The advantage of Doubly-Fed Induction Generator over Squirrel-Cage Induction Generator include robustness, solid rotor mechanical and electrical simplicity, low production price, high speed performance, much reduced distortion, excellent starting torque for high inertia, and etc. Its major drawback, on the other hand, can be considered as requirement of stator to receive magnetizing current. Increasing the cost by widening speed range around the synchronous speed and inevitable need for slip rings are of the other disadvantages of DFIG.

### Materials and methods

To achieve the research objectives in order to investigate dynamic interaction between large wind turbines/farms and power networks and also design and evaluation of the behavior of wind turbine under normal and disturbance conditions, two mathematical models of aerodynamic (rotor) and DFIG were developed. In the study, aerodynamic (rotor) model is based on torque coefficient  $C_q$  look up table which is according to BEM method. DFIG model is developed in two sub models including generator and power converters. In generator model, electrical dynamic equations of wound rotor induction generator are provided in two different models (fifth-order model and third-order model) by choosing synchronous rotating reference frame.

### Discussion and results

#### Aerodynamic Model

Standard aerodynamic program typically used B.E.M method. Since this newly developed toolbox has been focused on increasing simulation speed of all models, the use of such standard aerodynamic program requiring much of time to simulate wind farms with several wind turbines is therefore not attractive. Therefore, a simplified aerodynamic model is used instead when the focus is only on the effect in the power, the aerodynamic torque  $T_{wtr}$  developed on the main shaft of a wind turbine (Hansen et al, 2007). This simplified aerodynamic model is based on the aerodynamic power coefficient  $C_p(\lambda, \beta)$  or on the aerodynamic torque coefficient  $C_q(\lambda, \beta)$ , look up tables.

Thus,  $T_{wtr}$  can be modeled in steady state operating point, based on steady state power/torque coefficient by one of the following relations:

$$T_{wtr} = \frac{P_{wtr}}{\omega_{wtr}} = \frac{1}{2} \frac{P_{AIR} \cdot \pi \cdot R^2 V_i^3 C_p(\lambda, \beta)}{\omega_{wtr}} \quad (1)$$

$$\text{OR } T_{wtr} = \frac{1}{2} \cdot P_{AIR} \cdot \pi \cdot R^3 V_i^2 C_q(\lambda, \beta) \quad (2)$$

$$\text{Where } C_q(\lambda, \beta) = \frac{C_p(\lambda, \beta)}{\lambda} \quad (3)$$

To clarify the concept, Figure 4 presents the power coefficient  $C_p(\lambda, \beta)$  for few value of the pitch angle while the look-up table used in the model contains the power coefficient for value of



in stationary frame appear as DC quantities in synchronous reference frame. As a result, the equivalent circuit model of the induction machine in synchronous reference frame runs faster in simulator due to the DC nature of the impressed variables (Ackerman, 2005). Two different models of Wound Rotor Induction Generator (WRIG) used in DFIG scheme include the fifth-order model and the third-order model. The former includes all dynamic terms while in the later, the transient flux terms of the stator are neglected.

**Fifth-Order Model**

The electrical dynamic equations of Wound Rotor Induction Generator (WRIG) used for DFIG operation and described through the fifth-order model including all dynamic terms are presented. In the present model, the following conditions and assumption are considered (Ekanayake et al, 2003):

- 1) The stator current was assumed positive when following toward the machine.
- 2) The equations were derived in the synchronous reference frame using direct (d) and quadrature (q) axis representation
- 3) The q-axis was assumed to be 90° ahead of the d-axis in the direction of rotation.
- 4) The q component of the stator voltage used within the model is chosen to be equal to the real part of the generator busbar voltage obtained from the load flow solution that is used to initialize the model.
- 5) The high order harmonic components in the rotor injected voltages are neglected.

The voltage equations of the stator and the rotor of the machine in both d and q axes are given by (Bose, 2002):

$$V_{ds} = R_s \cdot i_{ds} - \omega_e \psi_{qs} + \frac{d}{dt} \psi_{ds} \tag{4}$$

$$V_{qs} = R_s \cdot i_{qs} + \omega_e \psi_{ds} + \frac{d}{dt} \psi_{qs} \tag{5}$$

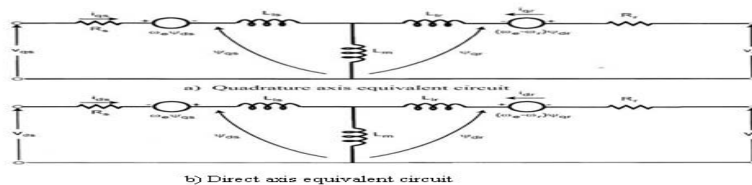
Since the rotor actually moves at speed  $\dot{\Lambda}_r$ , the d-q axis fixed on the rotor moves at a speed  $\dot{\Lambda}_e - \dot{\Lambda}_r$  relative to the synchronous rotating frame. Therefore, the rotor quantities should be modified as:

$$V_{dr} = R_r \cdot i_{dr} - (\omega_e - \omega_r) \psi_{qr} + \frac{d}{dt} \psi_{dr} \tag{6}$$

$$V_{qr} = R_r \cdot i_{qr} + (\omega_e - \omega_r) \psi_{dr} + \frac{d}{dt} \psi_{qr} \tag{7}$$

Where, V is the voltage, i is the current, R is the resistance,  $\psi$  is the flux linkage, subscripts r and s stand for stator and rotor, respectively and the subscripts d and q for direct and quadrature axis, respectively.

Figure 6 shows the equivalent circuit model of WRIG in synchronous reference frame that satisfy equations (4) to (7). In this equivalent circuit, the steady-state response of voltages and currents leads to constant DC signals representing the machine three-phase 120° shifted variables. Such an equivalent circuit model runs faster in simulator due to the DC nature of the impressed signals.



**Figure 6- Wound-rotor induction machine in synchronous rotating reference frame**



The flux linkages in equations (4) to (7) can be calculated from Figure 6 in terms of currents as:

$$\psi_{ds} = L_s \cdot i_{ds} + L_m \cdot i_{dr} \tag{8}$$

$$\psi_{qs} = L_s \cdot i_{qs} + L_m \cdot i_{qr} \tag{9}$$

$$\psi_{dr} = L_r \cdot i_{dr} + L_m \cdot i_{ds} \tag{10}$$

$$\psi_{qr} = L_r \cdot i_{qr} + L_m \cdot i_{qs} \tag{11}$$

Where  $L_m$  is the mutual inductance,  $L_s=L_{ls}+L_m$  is the stator self inductances and  $L_r=L_{lr}+L_m$  is the rotor self inductance. According to aforementioned, the zero quantities have been omitted in the above equations sine only balanced conditions are considered. Often, for compact representation, the machine model and equivalent circuits are expressed in the space vector complex form. Multiplying equation (4) by  $-j$  and adding with equation (5) gives (Bose, 2002):

$$V_{qs} - jV_{ds} = R_s(i_{qs} - ji_{ds}) + \frac{d}{dt}(\psi_{qs} - j\psi_{ds}) + j\omega_e(\psi_{qs} - j\psi_{ds}) \tag{12}$$

Or

$$\underline{V}_{qds} = R_s \underline{i}_{qds} + \frac{d}{dt} \underline{\Psi}_{qds} + j\omega_e \underline{\Psi}_{qds} \tag{13}$$

Similarly the rotor equations may be given as:

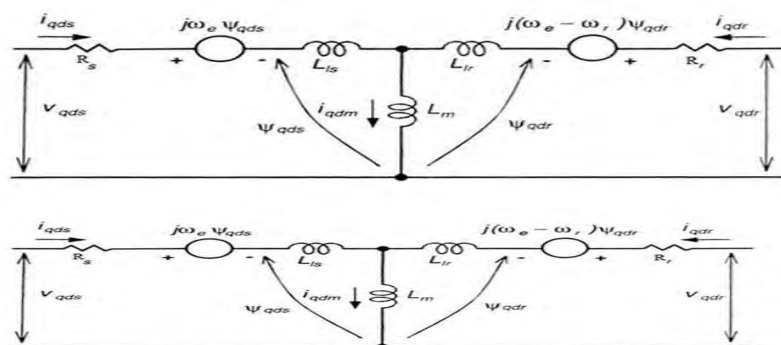
$$\underline{V}_{qdr} = R_r \underline{i}_{qdr} + \frac{d}{dt} \underline{\Psi}_{qdr} + j(\omega_e - \omega_r) \underline{\Psi}_{qdr} \tag{14}$$

In same way, flux linkage equations (8) to (11) can be given as following complex form:

$$\underline{\Psi}_{qds} = L_s \cdot \underline{i}_{qds} + L_m \cdot \underline{i}_{qdr} \tag{15}$$

$$\underline{\Psi}_{qdr} = L_r \cdot \underline{i}_{qdr} + L_m \cdot \underline{i}_{qds} \tag{16}$$

Figure 7 indicates the space vector complex equivalent model of wound-rotor induction machine; it is very compact representation that embeds the transient nature.



**Figure 7- Complex synchronous dq equivalent circuit for wound rotor induction machine**

If the speed  $\dot{A}_r$  is considered constant, the electrical dynamics of the machine are given by a fourth-order linear system; normally,  $\dot{A}_r$  cannot be treated as constant in the above equation. The equation of the motion of the generator can be rewritten as:

$$T_m = T_e + j \frac{d\omega_m}{dt} = T_e + \frac{2}{P} j \frac{d\omega_r}{dt} \tag{17}$$

Where  $j$  is the rotor inertia,  $p$  is the number of poles,  $T_m$  is the mechanical torque produced by the wind turbine and  $T_e$  is the electromagnetic torque of the generator which can be expressed in terms of d-q axis flux linkages and current as following complex form (Godoy Simoes et al, 2002):

$$T_e = \frac{3}{2} \cdot \frac{P}{2} \operatorname{Re} \left( j \underline{\Psi}_{qds} \cdot \hat{\mathbf{T}}_{qds} \right) = \frac{3}{2} \cdot \frac{P}{2} \operatorname{Re} \left( j \underline{\Psi}_{qdr} \cdot \hat{\mathbf{T}}_{qdr} \right) \quad (18)$$

Or it can be resolved in the synchronous reference frame as:

$$T_e = \frac{3}{2} \cdot \frac{P}{2} (\Psi_{ds} \cdot i_{qs} - \Psi_{qs} \cdot i_{ds}) = \frac{3}{2} \cdot \frac{P}{2} (\Psi_{dr} \cdot i_{qr} - \Psi_{qr} \cdot i_{dr}) \quad (19)$$

For a grid connected W.R.I.G, it is important to administer the generator active power transmitted through the stator and the rotor as following equation:

$$P = P_s + P_r \quad (20)$$

Where

$$P_s = \frac{3}{2} \operatorname{Re} (\underline{V}_{qds} \cdot \bar{i}_{qds}) = \frac{3}{2} (V_{qs} \cdot i_{qs} + V_{ds} \cdot i_{ds}) \quad (21)$$

or

$$P_r = \frac{3}{2} \operatorname{Re} (\underline{V}_{qdr} \cdot \bar{i}_{qdr}) = \frac{3}{2} (V_{qr} \cdot i_{qr} + V_{dr} \cdot i_{dr}) \quad (22)$$

The amount of active power can be found from following equation:

$$Q = Q_s + Q_r \quad (23)$$

Where

$$Q_s = \frac{3}{2} \operatorname{Im} (\underline{V}_{qds} \cdot \bar{i}_{qds}) = \frac{3}{2} (V_{qs} \cdot i_{ds} - V_{ds} \cdot i_{qs}) \quad (24)$$

or

$$Q_r = \frac{3}{2} \operatorname{Im} (\underline{V}_{qdr} \cdot \bar{i}_{qdr}) = \frac{3}{2} (V_{qr} \cdot i_{dr} - V_{dr} \cdot i_{qr}) \quad (25)$$

It should be mentioned that in case of D.F.I.G, the reactive power in equation (23) is not necessarily equal to the reactive power fed in to the grid which is the quantity that must be used for the load-flow solution. It depends on the control strategy for the grid side of the power converter feeding the rotor winding. It does not apply to the active power. Even though, the converter can generate or consume reactive power it cannot generate, consume or store active power. Hence, the expression for  $P$  in equation (20) gives the total active power generated by D.F.I.G and injected to the grid, apart from the converter's efficiency, which can be incorporated by multiplying the last two terms of this expression (i.e,  $V_{qr} \cdot i_{qr}$  and  $V_{dr} \cdot i_{dr}$ ) with the assumed converter efficiency. In other words, all active power fed in to or drawn from the rotor windings will be drawn from or fed in to the grid, respectively (Ackerman, 2005). Equations (13), (14), (17) and (18) give the complete fifth-order and non-linear model of the electro-mechanical dynamics of W.R.I.G in d-q synchronous reference frame which is able to represent rotor and stator transients correctly.

### Third-Order Model

Sometimes, the transient flux terms of the stator of the machine represented by derivative terms are neglected. For balanced steady-state condition, the variables are constant in the synchronous rotating reference frame. Hence, the electric transients can be ignored by setting  $d/dt z_{ds}$  and  $d/dt z_{qs}$  to zero and through which the fifth-order model of machine mentioned above will be reduced to third-order model (i.e., so called reduced-order model) in stator voltage equations (4) and (5). The main reason is to reduce computation time by cancelling a number of differential

equations during simulation. Solving equations (8) to (11) for the currents and substituting the result with voltage equations (4) to (7) by applying the principle of ignoring stator transients, the voltage equations may be written in terms of flux linkages (fluxes as state variables) as following matrix form equation (Hansen et al, 2004).

$$\begin{bmatrix} V_{ds} \\ V_{qs} \\ V_{dr} \\ V_{qr} \end{bmatrix} = \begin{bmatrix} \frac{R_s L_r}{D} & -\omega_e & -\frac{R_s L_m}{D} & 0 \\ \omega_e & \frac{R_s L_r}{D} & 0 & -\frac{R_s L_m}{D} \\ -\frac{R_r L_m}{D} & 0 & \frac{R_r L_s}{D} & -(\omega_e - \omega_r) \\ 0 & -\frac{R_r L_m}{D} & (\omega_e - \omega_r) & \frac{R_r L_s}{D} \end{bmatrix} \cdot \begin{bmatrix} \Psi_{ds} \\ \Psi_{qs} \\ \Psi_{dr} \\ \Psi_{qr} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \dot{\Psi}_{ds} \\ \dot{\Psi}_{qs} \\ \dot{\Psi}_{dr} \\ \dot{\Psi}_{qr} \end{bmatrix} \quad (26)$$

In order to avoid the computational problems due to algebraic loops, equation (26) should be written in state space form only in terms of the rotor fluxes as:

$$\begin{bmatrix} \dot{\Psi}_{dr} \\ 0 \\ 0 \\ \dot{\Psi}_{qr} \end{bmatrix} = \begin{bmatrix} \left( \alpha \cdot T_{rm} \cdot \sigma_r - \frac{\tau_{rm}}{\sigma_s} \right) & \left( \frac{\tau_{rm} \cdot \alpha}{\tau_{sr}} \cdot L_m + 1 \right) \omega_e - \omega_r \\ - \left( \frac{\tau_{rm} \cdot \alpha}{\tau_{sr}} \cdot L_m + 1 \right) \omega_e - \omega_r & \left( \alpha \cdot T_{rm} \cdot \sigma_r - \frac{\tau_{rm}}{\sigma_s} \right) \end{bmatrix} \cdot \begin{bmatrix} \Psi_{dr} \\ 0 \\ 0 \\ \Psi_{qr} \end{bmatrix} + \begin{bmatrix} \frac{\tau_{rm} \cdot \alpha}{\tau_{sr}} & \frac{\tau_{rm} \cdot \alpha}{\tau_{sr}^2} & 1 & 0 \\ -\frac{\tau_{rm} \cdot \alpha}{\tau_{sr}} & \frac{\tau_{rm} \cdot \alpha}{\tau_{sr}^2} & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} V_{ds} \\ V_{qs} \\ V_{dr} \\ V_{qr} \end{bmatrix}$$

Where  $\tau_{sr} = \frac{R_s L_r}{D}$ ,  $\tau_{rm} = \frac{R_r L_m}{D}$ ,  $\sigma_r = \frac{L_m}{L_r}$ ,  $\sigma_s = \frac{L_m}{L_s}$  and  $\alpha = \frac{1}{1 + \left(\frac{\omega_e}{\tau_{sr}}\right)^2}$  (27)

Based on rotor fluxes and input voltages, the stator fluxes can be obtained using the following matrix:

$$\begin{bmatrix} \Psi_{ds} \\ 0 \\ 0 \\ \Psi_{qs} \end{bmatrix} = \begin{bmatrix} \alpha \cdot \sigma_r & 0 & \frac{\alpha}{\tau_{sr}} L_m \omega_e \\ 0 & 0 & 0 \\ -\frac{\alpha}{\tau_{sr}} L_m \omega_e & 0 & \alpha \cdot \sigma_r \end{bmatrix} \cdot \begin{bmatrix} \Psi_{dr} \\ 0 \\ 0 \\ \Psi_{qr} \end{bmatrix} + \begin{bmatrix} \frac{\alpha}{\tau_{sr}} & 0 & \frac{\alpha}{\tau_{sr}^2} \omega_e \\ 0 & 0 & 0 \\ -\frac{\alpha}{\tau_{sr}^2} \omega_e & 0 & \frac{\alpha}{\tau_{sr}} \end{bmatrix} \cdot \begin{bmatrix} V_{ds} \\ V_{qs} \\ V_{dr} \\ V_{qs} \end{bmatrix} \quad (28)$$

Since flux linkages have been selected as state variables, the electromagnetic torque  $T_e$  of the generator is most conveniently stated as:

$$T_e = \frac{3}{2} \frac{P}{2} \frac{L_m}{D} (\Psi_{ds} \Psi_{qr} - \Psi_{dr} \Psi_{qs}) \quad (29)$$

But mechanical equation is the same in the fifth order model.

Figure 8 shows the Simulink model of the reduced order model of the induction machine based on equations (27) and (29).

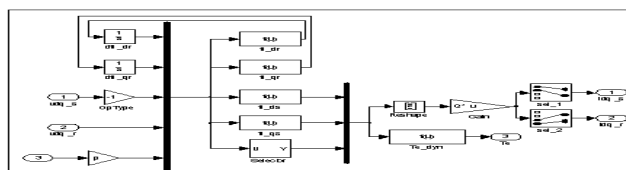


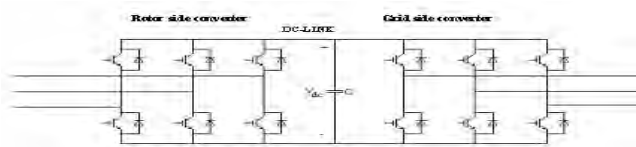
Figure 8- Simulink implementation of the reduced order model for the induction Machine

**Power Converters Model**

The power converters used in DFIG are usually realized by two identical self commutated Pulse-Width Modulated (PWM) circuits Voltage Source Converters (VSCs) which are connected back-to-back via DC-link and controlled independently in order to enable DFIG to transport power in both directions (Figure 9). As shown in the figure, each of the converters is built by six valves with turn-off capability and six anti parallel diodes. The valves are typically realized by IGBT because they allow higher switching frequencies (in the range of 2 to 20 KHZ) compared with classical GTOs.

According to the figure shown, the use of PWM technique realized by various methods involving selected harmonic elimination PWM, triangular carrier-based Sinusoidal PWM (SPWM), space vector PWM, random PWM, hysteresis band current control PWM and minimum ripple current PWM, permits the effective AC output voltage control and optimizes the harmonics (eliminating the low frequency harmonics) by performing multiple switching within the converter with the constant dc voltage (Krause et al, 2013).

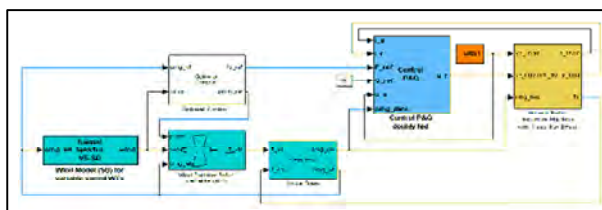
In fact, pulse-width modulator translates the modulation waveforms of variable frequency and magnitude into a train of switching pulses for the inverter/converter.



**Figure 9- Structure of back-to-back voltage source converter**

**Simulation Result**

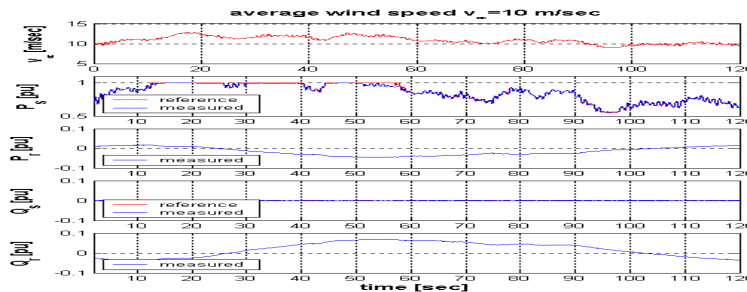
To evaluate the dynamic response of control algorithm using Matlab/Simulink toolbox (v2.0), Riso national laboratory and Aalborg university-Denmark have conducted studies on the control of active and reactive power for a variable speed 2 MW wind turbine using doubly fed induction generator (DFIG). Figure 10 presents Simulink diagram of the system.



**Figure 10- Simulink diagram of a 2 MW wind turbine using DFIG**

The simulation structure entails wind model, drive train model, a DFIG model written in synchronous reference frame, control block for active and reactive power (P&Q), and optimal control of the entire system (Hansen, 2007). In the simulation, it is assumed that the VSC model and its control model is condensed and lumped together in a unique module, therefore, the voltage source control impacts on the rotor of the machine is represented as a direct output of the VSC. In order to analyze the control of the active and reactive power for this system, a wind time series with an average value of 10 m/sec has been used. The synchronous speed of the machine has been considered as the base value for speed while the rated power of the machine is the base of the

active and reactive power (Hansen, 2007). Figure 11 shows the simulation results in terms of the wind time series, active and reactive power, both for the stator and the rotor circuit.



**Figure 11-Simulation results for a variable speed wind turbine with DFIG**

According to the simulation results, it can be indicated that at high wind speed, the stator power is limited at rated value while at lower wind speed wind turbine is operated at optimum power efficiency in order to obtain optimal power output. The reference of the stator reactive power and the measured one are zero in the entire simulation horizon. Since the wind speed has been acquired with the inherited sample time from simulation (0.05 sec) and used in the control algorithm, the reference for the stator power is not so smooth and the produced active power follows identically this reference. In a real system, the wind speed is acquired with a bigger sample time and some calculation are performed in order to find the average wind speed at each 1 min. Due to filtering the wind speed, the reference is much smoother and the output power will not exhibit this fast variations. It has been omitted the averaging block of the wind speed in order to study the dynamics performances of the control loops (Hansen, 2007).

The obtained results of the simulation are consistent with the real behavior of system. The accuracy of results can be more increased by using some blocks (e.g., control system blocks) developed in this dissertation.

### Conclusion

The present study attempted to investigate various aspects of variable speed wind turbine driving Double Fed Induction Generator (D.F.I.G) as well as develop its components` mathematical models including aerodynamic model and DFIG model through new matlab/simulink software. The so called models were developed due to the high growing penetration level of this scheme into power networks as well as the necessity of examining the influence of large wind turbines/farms in dynamic behavior of power networks and also designing and evaluating the behavior of each wind turbine under normal and disturbance conditions.

Aerodynamic (rotor) model was introduced based on torque coefficient  $C_q$  look up table derived according to BEM method. DFIG model was also developed in two sub models including generator and power converters. In generator model, by choosing synchronous rotating reference frame, the electrical dynamic equations of wound rotor induction generator (voltage differential equations, flux equations, the torque and motion equations) was provided in two different models involving fifth-order model and the third-order model. The former includes all dynamic terms while the later ignored the transient flux terms of the stator.

As the simulation result revealed, the stator power is limited at rated value at high wind speed but wind turbine is operated at optimum power efficiency at lower wind speed to gain optimal power output, indicating a good agreement with the real behavior of the system.



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