Presenting a Fuzzy Model for Fuzzy Portfolio Optimization with the Mean Absolute Deviation Risk Function

Mehdi Alinezhad Sarokolaei[1], Heydar Mohammadzadeh Salteh[2], Azadeh Edalat[3]

[1]Department of Accounting , Tabriz Branch, Islamic Azad University, Tabriz, Iran; [2]Department of Accounting , Marand Branch, Islamic Azad University, Marand, Iran; [3]Department of Accounting , Marand Branch, Islamic Azad University, Marand, Iran

Abstract

The main purpose of this paper is portfolio optimization with the use of fuzzy method based on the mean absolute deviation risk function in firms listed in Tehran Stock Market. In the present research, for the purpose of fuzzy portfolio optimization the stock portfolio Value at Risk criterion and for calculation of this value the parametric method and for fuzzy optimization also the Hybrid intelligent algorithms (genetic algorithms and neural networks) have been used. For selecting the portfolio with 15 during the research time span (2005-2011) fuzzy optimization based on the following six criteria were used including Asymmetric Value at Risk, Symmetric Value at Risk , Interval Value at Risk (interval of 5%-95%), Interval Value at Risk (interval of 10%-90%), and Normal Value at Risk. Since the calculated probability ratio statistic Kupiec based on fuzzy optimization for the 6 above mentioned models is larger than the obtained critical value from chi-square distribution at the confidence level of 95%, the research hypothesis stating that the application of fuzzy optimization method improves the efficiency of portfolio in the actual world problems with lack of certainty was confirmed. Also, the results of the Kupiec probability ratio statistic indicate that the model of value at risk based on the mean absolute deviation risk function (MVAR) is more successful and have less failure comparing to other models, hence; the research hypothesis stating that fuzzy variables have a higher ability in modeling asymmetric uncertainties in financial domains is also confirmed.

Keywords: portfolio optimization, the mean absolute deviation risk function, fuzzy optimization.

Introduction

The rapid advances in computer technology have caused the investment professional management to transform rapidly. The investor or the manager with the use of computer can access the detailed data and information regarding all the active companies in all the market sectors. The rapid advancement of computers and software have provided the possibility for individuals to use complex and advanced financial models on a daily basis. Therefore, the necessity of having access to efficient tools, which have been designed and developed with the use of this new technologies and based on the new investment theory, if felt intensely which can contribute to the decision making process of the investor society, investment analysts and the stock market which have a high level of expertise as well. These tools can help the managers and experts to obtain an extraordinary productivity. Also, we can use these tools and methods for the production and supply of the new financial products and also the identification and attraction of customers. In other words, with a deep understanding of these tools and methods the border line between success and failure in the path of investment can be determined (Haugen, 1993). As much as these tools are more exact and faster, they can help the optimized investment decision making process and the investor managers in their investment management in a better way.

Investment management includes two main topic of security analysis and portfolio management. Security analysis includes the estimation of every single investment advantages, while; portfolio management includes the analysis of investments and the management of a series of investments (Raee and Pooyanfar, 2010).

Three main elements have a key role in each profitable and successful investment management

Corresponding author:

which are: price prediction which states what kind of decisions should be made (sale and purchase), determines the time of trade and eventually the management of the amount of investment (Murphy, 1999), which all more than ever point to the importance of portfolio optimization.

Portfolio optimization refers to selecting the best combination from the financial assets which maximized the investment return and minimizes the risk of it as much as possible. The underlying idea of modern portfolio theory is that if investment will be made in a combination of assets which are not completely correlated with each other, the risk of those assets will neutralize each other; and therefore; a fixed return with a lower risk can be obtained. However; one of the most important challenges in choosing a criterion is an appropriate risk. So far, multiple and various risk criteria has been presented such as: variance (Markowitz, 1959), Value at risk (Linsmeier & Pearson, 1996), conditional value at risk (Rockafellar & Uryasev, 2000) and so on.

Since regulating the global economy and disposal of the shocks to it is possible due to the flexibility and strength of the advanced markets, these markets (including both centralized and decentralized) possess a huge part of the production, commercial and financial activities of humans (Azarkhsh, 2004). It is in such a way that we can say that today throughout the world, an extensive volume of the capitals are traded through stock market and the national economy of every country is related to its stock market performance and takes influence from it at an extensive level. In addition to it, recently, this type of market has turned into investment tools not only for professional investors but also for beginners. Therefore; this issue not only is related to macro-economic parameters, but also it affects the everyday life of humans and therefore they form such mechanisms which have important and direct social impacts.

Investment development from one side has caused the attraction of inefficient investment and has directed them toward the productive sectors of the economy and on the other hand considering the orientation of the investors (based on risk and return), the investments will be directed toward the industries with higher profit and lower risk and this would lead to the optimized allocation of the limited resource and this is the main aim of creating capital markets. Considering the developments occurred in the today's world, especially in developing countries which are facing serious threats, these countries for solving their economic problems need appropriated solutions for making a better use of their God granted facilities and wealth and one of the best solution is to develop investment (Tehrani and Noorbakhsh, 2009). On the other hand, in countries with high cash flow in the hand of people, with development and extension of the capital markets and attracting these capitals we can prevent economic damages and push economy toward Excellency (Yari, 2008).

The return resulting from investment has a special important for investors, because all the investment activities are performed in line with obtaining return. Return assessment is the only reasonable way (prior to risk evaluation) that investors can do for comparing the substitute and different investments (Tehrani and Noorbakhsh, 2009).

Portfolio optimization is the basis of investment in the environments of capital market with high level of uncertainty and turbulence and multiple papers have studied and presented solutions for improving the efficiency of portfolio. In this paper we deal with portfolio fuzzy optimization in which the return rate of investment is shown in fuzzy data. In the presented fuzzy optimization approach for portfolio the Zadeh's extension principle and the mean absolute deviation risk function are used. For calculation of the upper and lower bounds of the return rate of the whole problem of the fuzzy optimization portfolio, a pair of two-level program is formulated and presented. Based on the duality Theorem and application of variable mapping method, the above mentioned pair of two-level mathematical programs is changed into a pair of one-level normal linear program and therefore they can be used and solved. Eventually, in terms of one example in which the data from Tehran Stock Market is used, the presented framework for fuzzy portfolio optimization is implemented and its results will be discussed. Therefore; the main aim of this research is to present a fuzzy model for portfolio optimization with the use of mean absolute deviation risk function and for more clarity of the subject at hand a few instances from empirical studies conducted in this field will be presented and after that the research method, findings and conclusion will be presented.

Research background

Sajaddi *et al.* (2012) have presented a method for resistant portfolio optimization with considering the cardinality constraints. They have considered the uncertainty at the average of the expected return rate of capital and in a form of uniform symmetric distribution which is in fact equal with the set of hypercube uncertainty. Taking note of the fact that Cardinality constraints are non-linear, the genetic algorithm has been used for problem optimization. Although, the two above mentioned types of the uncertainty sets are repeatedly used, there is two main problem and issue about them. First, data collection for determination of the exact bounds and the boundaries of the uncertain "unknown but limited" sets of is in practice difficult. Sometimes, only a probable distribution can be obtained from the historical data. In this case, the uncertain "unknown but limited" sets in practice have boundaries with fluctuation and change. Second, the symmetry assumption of the distribution in so many of the applications, especially in financial systems modeling is restricting, which in most of them mostly from beforehand we know that the distributions are asymmetric.

Gerkez *et al.* (2010) have conducted a study with the title of "portfolio selection and optimization with the use of genetic algorithm based on different definitions of risk" that in financial subjects, portfolio can be seen as combination or a set of investments which is managed by an institute or a person. Portfolio selection for the purpose of profit or return maximization is one of the main concerns of the investors in the financial markets. The current available methods in optimized portfolio selection don't possess the necessary and required efficiency and hence for solving this problem, creative algorithms have received so much of attention. Genetic algorithm is a creative algorithm which can perform the portfolio optimization problem successfully with considering the different levels of risk. The aim of the present research is to select and optimize a portfolio based on different levels of risk. For achieving this goal tow genetic algorithm have been developed. In the development process two basic models have been considered: Markowitz's mean – variance model and average –semi-variance model. For the purpose of achieving more efficiency, some restriction related to the actual world have been added to the developed algorithms and the findings of the study indicate that the developed genetic algorithm has high level of stability and optimization in different runs. Taking note of the obtained results, it is determined that there is no significant difference in application of these two models (mean-variance model and mean-semi-variance model).

Amiri and Khaloozadeh (2006) have conducted a research with the title of "Determination of optimized portfolio in Iran stock market based on theory" and have stated that so many studies have been conducted during the recent years for development of risk management methods based on the theory of Value at Risk. In this study with the use of genetic algorithm (GA) an optimized portfolio is obtained which has the maximized profit/ return, while at the same time it has a restriction on the portfolio risk. VaR as well has been considered as the risk estimation criterion. This criterion simply and only with one numeric models the market risk. GA method is among one of the numerical optimization algorithms which have been inspired from natural genetic and evolution process in the nature. The main advantage of these algorithms is their so much high flexibility in dealing with complex problems and the fact that they do not need any mathematical conditions such as continuity and differentiable functions. Stimulation for has been performed for a portfolio consisting of 12 different firms in Tehran stock market. The obtained results indicate to the efficiency of the method of market risk modeling based on the theory of value at risk and the genetic algorithms optimization method in obtaining the optimized weights of the portfolio with considering the restriction on risk.

Variable under study in the present research in terms of a conceptual model and a description of the way of study and measurement of these variables

Assume that n is the various capitals available for investment and we want to invest the primary capital of M0 on a subset of the above items. Assume that the decision variables are x_j, j=1,2,...,n, which show the ratio of the allocated capital to asset j to the total assets. Also, we assume the restraints of $\sum_{j=1} x_j = M_0$ and $x_j \geq 0$.

Consider R_j as a random variable which shows the return rate of stock j in any time interval. Mathematical expects portfolio return in each interval will be calculated as per the following:

$$r(x_1,\ldots,x_n) = E\left[\sum_{j=1}^{n} R_j x_j\right] = \sum_{j=1}^{n} E\left[R_j\right] x_j$$

In general, distribution of investment on a set of assets instead of focusing it on a specific asset is performed for the purpose of avoiding risk. Markowitz has used the standard deviation of portfolio return as a risk indicator.

$$\sigma(x_1,\ldots,x_n) = \sqrt{E\left[\left\{\sum_{j=1}^{n} R_j x_j - E\left[\sum_{j=1}^{n} R_j x_j\right]\right\}^2\right]}$$

A reasonable investor may be interested in achieving a certain amount of portfolio return with the minimum risk. Markowitz has presented the problem of portfolio selection in the form of a two rank optimization problem as per the following:

$$V = \min \sum_{i=1}^{n} \sum_{j=1}^{n} \sigma_{ij} x_i x_j$$

$$\text{s.t.} \sum_{j=1}^{n} x_j = M_0,$$

$$\sum_{j=1}^{n} r_j x_j \geq R_0,$$

$$0 \leq x_j \leq U_j, \quad j = 1, 2, \ldots, n$$

where, R_o is the return rate in the currency under study, r_j and U_j are the expected return rate and the up limit of the investment on asset j. in addition, σ_{ij} is the standard deviation of portfolio return.

In order to decrease the load of calculations and costs of management and transfer of stock as well as the cut-off effect, Cono and Yamazaki [7] have presented a mean absolute deviation function as the replacement for standard deviation risk function of Markowitz for the obtained return of portfolio in each period.

$$w(x) = E\left[\left|\sum_{j=1}^{n} R_j x_j - E\left[\sum_{j=1}^{n} R_j x_j\right]\right|\right]$$

They proved that the criteria of mean absolute deviation and standard deviation are basically equivalent, if the variables of (R_1, R_2, ... ,R_n) have a normal multi-variable distribution and presented the portfolio optimization problem:

$$V = \min E\left[\left|\sum_{j=1}^{n} R_j x_j - E\left[\sum_{j=1}^{n} R_j x_j\right]\right|\right]$$

$$\text{s.t.} \sum_{j=1}^{n} x_j = M_0,$$

$$\sum_{j=1}^{n} E\left[R_j\right] x_j \geq R_0,$$

$$0 \leq x_j \leq U_j, \quad j = 1, 2, \ldots, n$$

With the assumption of having access to the historical data for each of the assets of the T previous years which provides access to price volatilities and the dividend payment – the return rate of each of the assets of the former assets can be estimated. r_{jt} is considered as the value of the realized variable of R_j during the t^{th} interval. It is obvious that the values of r_{jt} is not constant and can have significant changes from one year to the other and can take positive, negative and zero values. Hence; for assessment of the potential of the asset investment of r_{jt} to the paid money amount for it, r_j can be written as the following:

$$r_j = E\left[R_j\right] = \frac{1}{T}\sum_{t=1}^{T} r_{jt}$$

Therefore, w(x) can be estimated as per the following:

$$E\left[\left|\sum_{j=1}^{n} R_j x_j - E\left[\sum_{j=1}^{n} R_j x_j\right]\right|\right] = \frac{1}{T}\sum_{t=1}^{T}\left|\sum_{j=1}^{n}\left(r_{jt} - r_t\right)x_j\right|$$

Eventually, the linear portfolio optimization model can be rewritten as below:

$$V = \min \frac{1}{T}\sum_{t=1}^{T}\left|\sum_{j=1}^{n}\left(r_{jt} - r_t\right)x_j\right|$$

$$\text{s.}t. \sum_{j=1}^{n} x_j = M_0,$$

$$\sum_{j=1}^{n} r_j x_j \geq R_0,$$

$$0 \leq x_j \leq U_j, \quad j = 1, 2, \ldots, n$$

With making use of the techniques for transforming the linear functions, the above model can be rewritten as the below linear program:

$$V = \min \frac{1}{T}\sum_{t=1}^{T} u_t$$

$$s.t. \quad u_t + \sum_{j=1}^{n}\left(r_{jt} - r_t\right)x_j \geq 0,$$

$$u_t - \sum_{j=1}^{n}\left(r_{jt} - r_t\right)x_j \geq 0,$$

$$\sum_{j=1}^{n} x_j = M_0,$$

$$\sum_{j=1}^{n} r_j x_j \geq R_0,$$

$$0 \leq x_j \leq U_j, \quad j = 1, 2, \ldots, n$$

Presence of uncertainty in prediction of the return rate of any asset in the future causes the predicted value for these parameters to be inaccurate. Assume r_j and r_{jt} can be estimated in an approximate way and can be represented with convex fuzzy sets of $\tilde{R}_j$ and $\tilde{R}_{jt}$. Assume that $\mu_{\tilde{R}_j}$ and $\mu_{\tilde{R}_{jt}}$ indicate their membership function. Taking note of these values we will have:

$$\tilde{R}_{jt} = \left\{r_{jt}, \mu_{\tilde{R}_{jt}}\left(r_{jt}\right) \mid r_{jt} \in S\left(\tilde{R}_{jt}\right)\right\},$$

$$\tilde{R}_j = \left\{r_j, \mu_{\tilde{R}_j}\left(r_j\right) \mid r_j \in S\left(\tilde{R}_j\right)\right\}$$

In which $S\left(\tilde{R}_j\right)$ and $S\left(\tilde{R}_{jt}\right)$ are the rules $\tilde{R}_j$ for $\tilde{R}_{jt}$ and . Also, we have:

$$\tilde{R}_j = \sum_{t=1}^{T}\frac{\tilde{R}_{jt}}{T}$$

Where, the return rate of the assets $\tilde{R}_{jt}$ is fuzzy number and the parameters of $\tilde{R}_j$ and the value of the objective function will be as well fuzzy. Taking note of these assumptions, the portfolio optimization problem will turn into the below linear planning model with fuzzy parameters:

$$\tilde{V} = \min \sum_{t=1}^{T} \frac{u_t}{T}$$
$$s.t. \quad u_t + \sum_{j=1}^{n} \left(\tilde{R}_{jt} - \tilde{R}_j\right) x_j \geq 0,$$
$$u_t - \sum_{j=1}^{n} \left(\tilde{R}_{jt} - \tilde{R}_j\right) x_j \geq 0,$$
$$\sum_{j=1}^{n} x_j = M_0,$$
$$\sum_{j=1}^{n} \tilde{R}_j x_j \geq R_0,$$
$$0 \leq x_j \leq U_j, \quad j = 1, 2, \ldots, n$$

Without losing the problem generality, all the parameters of $\tilde{R}_j$ and $\tilde{R}_{jt}$ will be considered as convex fuzzy values. Since $\tilde{V}$ is a fuzzy value instead of an absolute value the development principle of Zadeh is used first to change the problem into a linear planning problem and then to solve it.

Research methodology

The present research is applied from aim point of view and is descriptive from the data collection method point of view.

Population and sample

The population of the present research includes all the active firms listed in Tehran Stock Market during the time span of 2005 – 2011 and the research sample has been selected with the use of elimination method and based on the following criteria:

1. For sample homogenization during the years under study, the firm should have been listed in Tehran Stock Market before 2005;

2. For increasing the power of comparison, the financial year of the firms should be ended by the Month of Esfand (last month of solar year) and shouldn't have changed their financial years during the time period of the research;

3. The information related to their audited financial statements should be available for the period of research;

Data collection tools

The data collection method used in this research is bibliographical and field methods and for this purpose first we have referred to the available resources (libraries, theses, articles, internet) for presenting the theoretical framework of the research and the efficiency, strengths, weaknesses of the existing methods, hypotheses and achievements of each method and the facing challenges in combining the existing methods for achieving higher capacities in portfolio fuzzy optimization has been studied. In the second section, with the use of field method and with reference to the site of stock market as well as using the available data bank, the required data for the research period has been collected and this data has been used for analysis of the research aims.

Research findings

In the present research we seek to perform fuzzy optimization on stock portfolio based on the criterion of portfolio Value at Risk with the use of mean absolute deviation function in the firms listed in Tehran Stock Market. In different studies, the value at risk has been normally calculated at the confidence levels of 95% (5% error level) and 99% (1% error level) and with the use of two parametric and non-parametric methods. In this research we have calculated it at the confidence levels of 95% and 99% with the use of parametric method.

For fuzzy optimization the hybrid intelligent algorithms (genetic algorithms and neural network) has been used. The used genetic algorithm in this research is a single-stage algorithm. The used selection technique is the Roulette Wheel. The number of generations is 2000 and the population of each generation is 20. This algorithm has been written with the use of MATLAB software.

The value at risk has been presented based on each of the 6 below criteria. In this research for the selection of portfolio of 15 stocks during the 7 periods of research (2005 to 2011), the fuzzy optimization method has been implemented as per the following criterion:

1. Asymmetric Value at Risk (ARVaR)

2. Symmetric Value at Risk (SRVaR)

3. Interval Value at Risk (interval of 5% to 95%) IRVaR $_{5\%\text{-}95\%}$

4. Interval Value at Risk (interval of 10% to 90%) IRVaR $_{10\%\text{-}90\%}$

5. Normal Value at Risk (NVaR)

6. Value at Risk based on the mean absolute deviation function (MVAR)

In order to estimate the reliability of the fuzzy optimization models based on asymmetric value at risk, symmetric value at risk, interval value at risk (period of 5% to 95%), interval value at risk (period of 10% to 90%), normal value at risk (NVaR) and value at risk based on the mean absolute deviation function (MVAR), the Kupiec failure probability ratio test has been used. In this test when the calculated value of LR, which is based on the model data, is larger than the critical value extracted from the chi-square distribution; therefore, in the given confidence level it can be claimed that that the model's predicted error percentage will be maximum equal to the determined error level of a and the model has an appropriated reliability.

Considering the research findings, since at the confidence level of 95%, the calculated LR based on fuzzy optimization for the five mentioned models is larger than the critical value extracted from the chi-square distribution, at confidence level of 95% we can claim that the model's predicted error level is maximum equal to the determined error level (a) and therefore, the model has an appropriated reliability. Therefore, the 2nd research hypothesis indicating to the fact that application of fuzzy optimization method improves the portfolio efficiency in the problems of the actual world with uncertainty is confirmed.

Table 1. Models' ability comparison

Model	Confidence level	Number of success	Number of failures
Asymmetric value at risk (ARVaR)	95%	1121	139
Symmetric value at risk (SRVaR)	95%	1033	227
Interval value at risk (5% - 95% interval) IRVaR $_{5\%-95\%}$	95%	1159	101
Interval value at risk (10% - 90% interval) IRVaR $_{10\%-90\%}$	95%	1172	88
Normal value at risk (NVaR)	95%	1084	176
Value at risk based on the mean absolute deviation function	95%	1134	126

In order to compare the ability of the above 6 models which are based on fuzzy optimization and based on the criterion of value at risk, the Kupiec failure probability number has been used. If the number of the success of a model in estimating the value at risk in a time interval will be larger than another model, that model has a higher prediction and measurement power. As we can see in table 1, the model of value at risk based on the mean absolute deviation function (MVAR) has less failures and more success comparing to other models. Hence, the 1st research hypothesis indicating that these fuzzy variables have a higher ability in modeling the existing uncertainties in the financial fields is also confirmed.

Conclusion

Portfolio management for realizing the investors' goals seeks to obtain profit and manage risk. An investor often seeks to have the highest amount of return at the lowest possible risk. Other limitations can also be present in a certain investment. A set of market limitations and investors preferences together with expected return and risk of the assets determines the applied strategy by the portfolio managers (Torrubiano & Suarez, 2008). Generally, two different underlying approaches are applied for assets management and acquiring the expected returns and risk of the investors which are: active portfolio management and passive portfolio management (Beasley & Meade, 2003). Passive portfolio seeks to obtain a return equal to the return of a portfolio with specific criterion, while active portfolio management indicate to the resource allocation based on an active strategy and contrary to passive management, its main and primary aim is not only to obtain a positive return but to obtain higher return than normal (extra). The meaning of extra return is to have a performance better than the specified indicator. This indicator is normally one of the existing indices in the stock market (Grinold & Kahn, 2000). Portfolio optimization is the basis of investment in the highly uncertain and turbulent environment of stock market and so many researchers have studied and provided some solutions for this problem. One of the most important topics in this field, is the uncertainty which prevails in this markets. Portfolio optimization based on inaccurate point estimates can be intensely misleading and the value of the objective function practically is worse than the calculated optimized value for that based on the nominal values of the parameters. Hence; consideration of a method for having a desirable value for the practical value of the objective function, even in spite of the inaccurate estimations, is so much necessary and can prevent major damages which can be caused as a result of not considering this matter. One of the most important topics in this field is the uncertainty which prevail these markets. Portfolio optimization based on the estimated point values for the values of different assets' return and the covariance between them can be highly misleading and it can cause undesirable results. Hence; considering a method for including these inaccuracies in the real world and modeling the inherent uncertainty of the predicted parameters is so much necessary and can prevent the occurrence of the major loss and damages that lack of attention to this matter can create. Using fuzzy logic as a method that has proved its efficiency in different applied fields in the past few decades can result in improvement of the efficiency of the formed portfolio in the situation of uncertainty. Portfolio optimization refers to the selection of the best combination of the financial assets in a way to maximize the investment return as much as possible and minimize the risk of investment as much as possible. The fundamental idea of the modern portfolio theory is that if we will invest in assets which are not completely correlated with each other, the risk of those assets will be neutralize with each other; therefore, we can have a constant return with a lower risk. Numerous experiences have shown that portfolio optimization based on inaccurate estimation of the parameters can be highly misleading and the objective function practically can be worse than the optimized calculated value based on these estimations. Hence, using a method such as fuzzy logic which includes the existing uncertainties in the process of the selection of portfolio is so much necessary.

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