European Online Journal of Natural and Social Sciences 2022; www.european-science.com
Vol.11, No 4 pp. 1212-1233
ISSN 1805-3602

Investigation of the Critical Thinking Process in Solving Non-Routine Mathematical Problems

Aiyub Aiyub[1&2*], D. Suryadi[1], S. Fatimah[1], Kusnandi Kusnandi[1]
[1]Department of Mathematical Education, School of Postgraduate, Universitas Pendidikan Indonesia, Bandung, Indonesia; [2]Department of Mathematical Education, Universitas Islam Negeri Banda Aceh, Indonesia
*E-mail: aiyub@upi.edu

Received for publication: 23 April 2022.
Accepted for publication: 06 June 2022.

Abstract

There have been many investigations of the critical thinking processes of junior high school students in solving math problems. However, related investigations based on the Theory of Didactic Situations of Mathematics (TDSM) have not been explored. Therefore, this qualitative research using Interpretive Phenomenological Analysis (IPA) was conducted to reveal the variety of students' critical thinking processes in solving non-routine problems of rational numbers. Data were obtained by providing non-routine problems and semi-structured interviews to get the necessary information. Participants were taken from class VIII in one of the State Islamic Junior High Schools in Banda Aceh City, Indonesia (n=15). Data were analyzed through data management, reading-memoing, describing-classifying-interpreting, and representing-visualizing. The results showed that Brousseau's didactic situation framework can help students in solving non-routine rational number problems. The critical thinking processes of students who solve problems with critical reflection are identified into six components of Ennis' thinking, namely identifying focus, identifying and evaluating relevant reasons, making inferences, assessing situations, providing clarity, and doing overview. The critical thinking process of students who solve problems with explicit reflection can be identified as the five critical thinking components of Ennis except doing an overview. While the critical thinking process of students who cannot solve problems only appears to identify focus and make conclusions. The results of this study are expected to be used as literature to develop students' critical thinking skills.

Keywords: Critical Thinking, Didactic Situations, Problem Solving, Rational Numbers

Introduction

Humans are inseparable from activities associated with evaluating and solving problems, decision-making, predicting, interpreting, and building causal relationships throughout their entire lives. In other words, they make assumptions, develop hypotheses, and try to find solutions by focusing on the reasons and outcomes of the thinking process (Ersoy & Başer, 2014). It includes the manner a certain problem is solved, interpretation of a mathematical symbol, proving a statement, determining whether or not it is flexible and relates to several concepts, etc. The characteristic of such mental action is described as a way of thinking (Harel & Sowder, 2005; Harel, 2008). According to Suryadi (2019), the way of thinking (WoT) facilitates understanding, which improves mental action. To think about something means to be aware of it and take necessary steps as well as consider the consequences (Dewey, 1910).

The educational objectives are to enhance students' thinking process to a higher and broader level that will contribute to academic achievement and future careers (Weiss, 1993). Critical thinking is an educational goal that is widely accepted by many people (Hitchcock, 2018). These skills have become an important competency for those in the new information era and global economic society (Mason, 2007; Biber et al., 2013). Fisher (2001) stated that critical thinking is widely perceived as a basic competency, similar to reading and writing.

International studies show that the mathematical ability of Indonesian students is still low compared to other countries. The 2015 Trends in International Mathematics and Science Study (TIMSS) study showed that Indonesia was ranked 45th out of 50 participating countries (Mullis et al., 2015). In the Programme for International Student Assessment in 2018," which was released in 2019, the Indonesian students took the 74th position out of the 79 PISA participating countries (OECD, 2019). Setiana et al. (2021) say Indonesian students can only master simple mathematics or easy problem solving but have not been able to solve complex problems and complex problems. Ansari et al. (2021) said that misconceptions and errors influenced students' failure to solve complex problems in making mathematical models. The results of this study are in line with the results of other studies regarding the low critical thinking skills of Indonesian students, such as (Akgun & Duruk, 2016; As'ari et al., 2017; Biber et al., 2013; Changwong et al., 2018; Basri et al., 2019; Ichsan, 2020; Aiyub et al., 2021). Kriswantoro et al. (2021) said that students still find it very difficult to do tests on critical thinking process skills.

To develop students' critical thinking skills, many studies have been carried out by developing various strategies or learning models, including problem-based learning (Kardoyo et al., 2020), inquiry learning (Adnan et al., 2021), E-learning with discovery learning models (Chusni et al., 2020). Setiana et al. (2021) use a mathematical learning model to stimulate critical thinking skills. The most effective and logical strategy for developing critical thinking skills is to facilitate by providing opportunities for students to discover and practice the skills themselves and give them the necessary feedback on their progress (Marcia, 1991). To be optimally effective, Marcia said that a learning framework is needed that supports differences in learning styles and abilities, allows for observation, interaction with, and internalization processes, and supports student interaction to help crystallize learning.

The study of literature on critical thinking processes has been discussed by (Dewey, 1933; Jenicek & Hitchcock, 2005; Hitchcock (2017; Hitchcock, 2018). Several researchers have investigated students' critical thinking processes in solving mathematical problems based on the level of students' mathematical abilities, such as (Fadhilah et al., 2021; Tohir et al., 2021; Rasiman, 2013). The critical thinking process of high school students (Fadhilah et al., 2021: Rasiman, 2013). Assessment of critical thinking processes in computer conferencing (Fahy, 2002).

All the research that has been done above only looks at the critical thinking process by using independent knowledge or mature knowledge (actual development) and has not considered the knowledge of students who are still in the maturation process (potential development). Vygotsky said that to see the development of children's knowledge as a result of learning, not only what has matured (actual development) but also needs to consider knowledge that is still in the process of maturation (potential development) known as the Zone of Proximal Development (ZPD) (Zaretskii, 2009). The Theory of Didactical Situations of Mathematics (TDSM) (Brousseau, 2002) framework is a model that can be used to analyze critical thinking processes systematically in solving problems according to Vygotsky's ZPD concept as proposed by (Marcia, 1991). Many studies related to the TDSM framework have been carried out such as (Artigue & Blomhøj, 2013; Nickels & Cullen, 2017; Strømskag 2017; Danisman & Guler, 2019; Suryadi, 2019b; Rudi et al., 2020).

The purpose of this study was to determine the diversity of students' critical thinking processes in solving non-routine rational number problems. This article is expected to be a literature to develop students' critical thinking skills.

Literature Review

Some studies define critical thinking as a process that aims to make rational decisions (Ennis, 1996b; Facione, 2015; Krulik & Rudnick, 1988; Lai, 2011). In this case, it is focused on purposeful variables or factors. Critical thinking aims to enable decision-making. Choy & Oo (2012) stated that it is the process of analyzing and making judgments about certain occurrences. The phrase "make judgments" is emphasized, explicitly implying that the decision is part of critical thinking. Facione (2018) states that this reflective process, also known as "critical thinking," cannot be separated from its goal, which is to form reasonable judgments about what needs to be believed or decided. Beyer (1985) stated that a student has certain criteria or benchmarks with respect to critical thinking and direction; therefore, it is necessary to either determine the basis for the decision or believe in something. Hitchcock (2017) distinguishes critical thinking and logical thinking in three main differences: going beyond a single argument, having a creative component represented by proposing and evaluating alternatives and selecting the best of them, and involving the critical appraisal of the evidence itself.

Ennis (2015b) reported five components of this process: basic and further clarification, basis for decisions, inference, and non-constitutional facilitation. In addition, Ennis provides a component of the critical thinking process as a critical thinking checklist with six components, which are abbreviated as Frisco, namely identifying focus, identifying and evaluating relevant reasons, considering inferences, assessing the situation, providing explanations, and conducting an overview (Ennis, 1996a; Hitchcock, 2017). Setiana et al. (2021) made a connection between the five components of critical thinking with the FRISCO critical thinking checklist from Ennis in which basic clarification for focus identification, building basic skills for identifying and evaluating reasons, inference, further clarification for assessing situation conclusions and provide clarity for terms and develop strategies and tactics for overview.

Theory of Didactic Situations of Mathematics (TDSM)

The Theory of Didactic Situations of Mathematics (TDSM) is an effective mathematical learning procedure involving the creation of an interactive session between the teachers and students. Brousseau (2002) reported that didactic situations are divided into four types, namely action, formulation, validation, and institutionalization. Furthermore, Brousseau stated that learning always starts with an active situation. This creates an opportunity for the child to apply previous experiences and knowledge. According to Tall (1999), the proper occurrence and encapsulation of this process facilitate the formation of new mental objects (Dubinsky, 2001). Brousseau described the situation as the substantial occurrence of abstraction due to mental actions. The process of interaction between students and teachers permits the determination of meaningful arguments, statements or claims, and representations which promote both the internal and external validation processes referred to as the final situation (Brousseau, 2002). The relationship between the TDSM situation (Brousseau, 2002) and the components of Ennis' critical thinking process can be described as shown in Table 1.

The learning sequence based on the TDSM framework consists of different phases, namely the adidactic phase, in which there is little or no intervention from the teacher, and the didactic phase, in which the teacher is more active and acts with didactic goals, i.e., students have to learn something (Rønning, 2021). Ideally, the first three phases in TDSM are adidactic, while the last phase is didactic. Ideally, the first three phases in TDSM are adidactic, while the last phase is didac-

tic. Nickels & Cullen (2017) say a child enters an adidactic situation through a process, or devolution stage, as in the taxonomy (1) pure-play, (2) preference transfer, (3) responsibility and causality transfer, (4) anticipation transfer, and (5) transfer of responsibility.

Table 1. Relationship of TDSM situation with critical thinking process components

Numb	TDSM	Critical Thinking Process Components
1	Action situation	Identify ocus of the problem
		Identify relevant reasons
2	Formulation situation	Evaluate relevant reasons
		Making inference
3	Validation situation	Assess the situation
		Provide clarity
		Do an overview

Strømskag (2017) designed a mathematics learning model based on TDSM in four phases, namely epistemological analysis, development of epistemological models, implementation, and institutionalization. Strømskag describes the four phases: 1) epistemological analysis consists of two components, namely knowledge analysis itself to identify possible epistemological obstacles and supports the search for underlying situations, and didactic analysis components to seek knowledge that might guide the design; 2) Development of an epistemological model consisting of three components: a model of target knowledge, a situation that retains meaning, and an environment of action situations, formulation and validation designed in such a way as to make students' knowledge development in a more explicit direction; 3) application, namely as an independent adaptation through the concept of adidactic situation and environment; and 4) institutionalization in which the teacher compares the contextualized knowledge (student solutions) with the scientific knowledge aimed at by the institution.

The process of forming (interpreting) the concept or image (meaning) of a didactic situation is closely related to the reflective inquiry. This thinking process involves the application of previous experiences in understanding the assumptions and implications of a problem. This process occurs at three levels: implicit, explicit, and critical reflections (Suryadi, 2019a). According to (Suryadi 2019), when an individual is faced with a problem and refers to personal experiences and knowledge to find an answer or solution, the process is known as implicit or a first-level reflection process. However, when faced with a problem that is not resolved by referring to one's personal experiences and knowledge, thereby causing the individual to seek information from external sources such as preliminary studies and discuss with several parties within the environment till the answer or solution is obtained. This process is known as an explicit or a second-level reflection process. Sometimes, the process of finding the solution to the problem at hand is usually deadlocked. In such a situation, the person engages in critical reflection, a reflective process that triggers human creative power till new answers, which are not based on prior explicit knowledge, are found.

Zone of Proximal Development (ZPD)

Suryadi (2011) stated that the Zone of Proximal Development (ZPD) is the distance between the actual and potential developments. The actual development is the accumulation of abilities independently obtained through the learning process (without the help of others) when solving problems. Meanwhile, potential development occurs as a result of interactions with teachers or knowledgeable students. The acquisition of ZPD requires support from teachers or capable individuals, known as scaffolding. Wood et al. (1976) described this process as a procedure that allows children to independently solve problems, perform tasks, or achieve certain goals. It also includes the process of adding a child to the learning process. Hartman (Van Der Stuyf, 2002) stated that the purpose of scaffolding is to ensure that students become independent and confident learners and problem solvers. The scaffolding approach proposed by Henningsen & Stein (1997) effectively promotes the development of students' higher-order thinking skills. Van de Pol et al. (2010) divide scaffolding into six types: feeding back, giving hints, instruction, explaining, modeling, and questioning. Pfister et al. (2015) provide a detailed explanation of Scaffolding from Van de Pol et al. as follows: 1) feeding back, namely giving feedback on student performance; 2) hints, namely giving instructions or suggestions; 3) instruction that is giving direction what to do or how something should be done; 4) explaining, namely providing more detailed information or clarification; 5) modeling, namely offering examples or behaviors to imitate; and 6) questioning, namely asking questions that require active linguistic and cognitive answers.

Methodology

Research Design

This type of research is qualitative research using an Interpretive Phenomenological Analysis (IPA) approach, which aims to understand the meaning and interpret a phenomenon based on human experience (Eatough & Smith, 2017). Where IPA is closely related to phenomenology and hermeneutics, which focuses on one's experience, as stated (Ricoeur, 1986), it is necessary to integrate the study of experience and understand the meaning because they are complementary. It was chosen to reveal the various meanings of students' critical thinking processes in solving non-routine mathematical problems with rational numbers based on a didactic situation theory framework (Brousseau, 2002). Prior research based on TDSM (Nickels & Cullen, 2017; Strømskag 2017; Danisman & Guler, 2019; Suryadi, 2019b; Rudi et al., 2020; Rønning, 2021).

Sample and Data Collection

The subjects of this study were 15 grade VIII students in one of the Islamic State Junior High Schools in Banda Aceh City, Indonesia. Of the 15 subjects who were given a problem-solving test based on the TSDM framework, two research subjects were selected to analyze the critical thinking process and the obstacles experienced by students in solving the problems given in accordance with the objectives of this study, namely 1) subjects who experience critical reflection, and 2) subjects who think critically that goes beyond a single argument with explicit reflection. Data was collected by giving tests and conducting interviews. Tests are given to collect data related to critical thinking processes and learning barriers experienced by students when solving given problems. Interviews were conducted to explore the results of the subject's work in solving problems. All data collection activities in this study were documented with audio-visual and sound recordings. The following indicators of critical thinking processes and descriptions were adapted from (Ennis, 1996a; Setiana et al., 2021).

Table 2. Indicators of Students' Critical Thinking Processes

No	Critical Thinking Process Components	Indicator of students' Critical Thinking Process
1	Identify Focus (F) of the Problem	Formulate a question
		Identify criteria for possible solutions
2	Identify and evaluate relevant reasons (R)	Identify relevant reasons
		Evaluating relevant reasons
3	Making inference (I)	Consider conclusions with logical reasons

No	Critical Thinking Process Components	Indicator of students' Critical Thinking Process
4	Assess the situation (S)	Considering alternative solutions
5	Provide clarity (C)	Give clarity to the language/terms used
6	Do an overview (O)	Found an error in the troubleshooting performed

Non-routine mathematical problem instruments are stated as follows:

> It is a known fact that a positive rational number less than one is expressed as an ordinary fraction in its simplest form. Therefore, assuming the product of the quantifier and denominator of the positive rational number is 7! = 7 x 6 x 5 x … x 1. Determine the number of positive rational numbers formed from the question?

The list of scaffolding questions that can be given to students if needed in solving non-routine rational number problems is attached in Table 3 below.

Table 3. List of Scaffolding Questions given to students in solving non-routine rational number problems

Critical Thinking Process Components	Scaffolding Question
Identify focus	What is known from the problem? Which is an example of a positive rational number less than 1? What is the value of the 7 factorial? What is asked from the question?
Identify relevant reasons	Which examples of positive rational numbers greater than 1 qualify, and which are not? Why don't $\frac{2}{2520}$, $\frac{3}{1680}$ and $\frac{4}{1260}$ qualify as positive rational numbers less than 1? What is the difference between the simple form of positive rational numbers ($\frac{1}{5040}$; $\frac{5}{1008}$) and the non-simple form of positive rational numbers ($\frac{2}{2520}$; $\frac{3}{1680}$)?
Evaluate relevant reasons	What are the prime factorization numbers of 5040? What is the strategy to get the factorized number from 5040? Can factorization be used to construct the requested positive rational number?
Making inference (I)	Which positive rational number less than 1 satisfies? How many positive rational numbers less than 1 satisfy? Is the reason used is correct?
Assess the situation	How can it be shown that the results obtained are correct? Are there other strategies that can be used to demonstrate that the conclusions drawn are correct? Is there a relationship between the number of subsets that can be constructed from a set and the number of positive rational numbers less than 1 that we can form?
Provide clarity	What does simple fraction mean?

Critical Thinking Process Components	Scaffolding Question
	What does FPB mean? What does prime factor mean? What does prime factorization mean?
Do an overview	Are all steps done correctly? Are all the terms used correct?

Analyzing of Data

The data collection results in the form of problem-solving recordings, answer sheets and scribbles, students' and teachers' interview documents, and observed information during the research were analyzed based on the stages developed by Creswell (2007). This includes data managing, reading-memoing, describing-classifying-interpreting, and representing-visualizing. Data management involves inputting information into computer files for analysis, transcribing recorded data and student interviews, and typing observation notes. Reading-memoing is the reading and interpreting of collected data and giving notes or memos in the margins of field, transcripts, or under photos to assist in the initial process of data exploration. Describing-classifying-interpreting is forming codes or categories representing the core data analysis. Consequently, detailed descriptions were constructed, themes or dimensions developed, and interpreted based on personal views or perspectives of the study. Representing-visualizing is depicting the data analysis results in the form of text, tables, or figures.

Results

Based on the results of the analysis of research data from 15 subjects who participated in this study, students' critical thinking processes in solving non-routine math problems can be grouped into three categories, namely first being able to solve problems with critical reflection; the second can solve the problem with the help of scaffolding; and lastly couldn't solve the problem. The complete results of the number of students in each category are presented in Table 4 below.

Table 4. Category of Students on Critical Thinking Process in Solving Problems

Category of Students in Critical Thinking Process		
Can Solve Problems		**Can't Solve the Problem**
Critical Reflection	**Explicit Reflection**	
2 student	10 student	3 student
13,33%	66,67%	20,00%

The following is an explanation of the data analysis for each component of the critical thinking process from the subject of S1 (Critical reflection), the subject of S2 (Explicit reflection), and the subject of S3 (Unable to solve the problem) based on a didactic situation.

Action Situation

The components of the critical thinking process related to the action situation are identifying the focus and identifying the relevant reasons. The following is a description of the research data from the two components of the critical thinking process.

Identifying focus. The indicator of identifying focus is that the subject can formulate questions or identify criteria for assessing possible answers. The action situation of the three subjects has not been able to formulate questions and identify criteria to assess the possible answers that are asked properly. This happened because S1 did not understand the factorial concept, while S2 and S3 did not understand the concept of rational and factorial numbers, so the three subjects questioned the concept. After receiving scaffolding assistance from their friends and researchers, both of them were able to respond well in formulating questions and identifying criteria for assessing possible answers. When the Researcher asks, "what are the criteria for judging the possible answers to that question?" The three subjects were able to give good answers, namely "...a positive rational number is less than 1, the product of the numerator and denominator is 5040...".

The statement of criteria for assessing possible answers above shows that the S1 action situation in identifying the focus of the problem only comes to an understanding of the concept of positive rational numbers while understanding the seven factorial concepts becomes a validation situation. Meanwhile, in S2 and S3, understanding the concept of rational numbers and factorial concepts cannot be done with action situations but must turn into validation situations. The following are the responses given by the three subjects in formulating questions from the given mathematical problems.

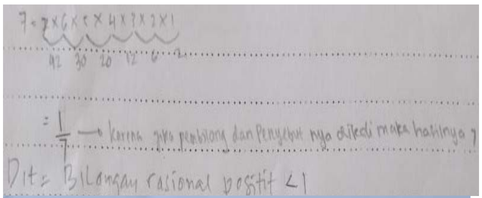
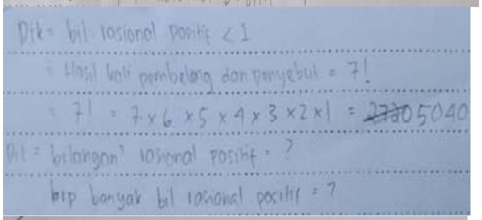
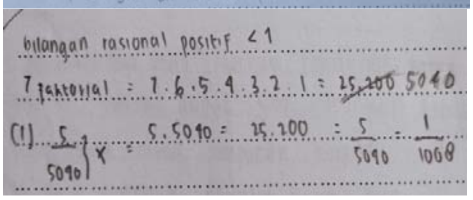
Subject	The Responses Given	Translation in English of Subject Response
S1		$7=\underbrace{7x\ 6}_{42}x\ \underbrace{5x\ 4}_{20}x\ \underbrace{3x\ 2}_{6}x\ 1$ $\frac{1}{7}\rightarrow$ because if the numerator and denominator are multiplied, the result is 7 Wanted: Positive rational number < 1
S2		Known: Positive rational number < 1 Product of numerator and denominator = 7! 7!= 7 x 6 x 5 x 4 x 3x 2 x 1= ~~2720~~ 5040 Asked: Positive rational number? How many positive rational numbers are there?
S3		Positive rational number < 1 7 factorial = 7.6.5.4.3.2.1= ~~25.200~~ 5040 $\frac{5}{5040}$ = 5.5040= 25.200 = $\frac{5}{5040}=\frac{1}{1008}$

Figure 1. The Response given by S1, S2, and S3 in identifying the focus

Identify relevant reasons. S1 and S2 subjects can identify relevant reasons under different conditions. S1 identifies reasons by providing examples and non-examples of positive rational numbers less than one requested from the given problem. Based on the samples and not the examples given, S1 can identify that the numbers from the numerator and denominator of simple fractions have the greatest common divisor (GCD) equal to 1 or relatively prime. The following is the data of the Researcher's (R) interaction with S1 in identifying the relevant reasons.

R : "Why does $\frac{2}{2520}$ not meet the positive rational number asked?"

S1 : "Because $\frac{2}{2520}$ is not an ordinary fraction, then if it is simplified, the numerator and denominator are not equal to 5040."

R : "Why are the numerators and denominators of common fractions relatively prime?"

S1 : "Simple fractions cannot be simplified anymore, meaning that their GCD is one or relatively prime, while non-simple fractions can still be simplified, meaning that their GCD is more than 1!"

Based on the data above, it shows that S1 can identify relevant reasons through action situations. While S2 identifies the relevant reasons by identifying the quotient of the number 5040 with a natural number, for example, 5040 divided by 1, the result is equal to 5040, so the fraction becomes $\frac{1}{5040}$, then 5040 is divided by 2, then the result is 2520, then the fraction becomes $\frac{2}{2520}$ and so on, so that S2 can construct ten examples of positive rational numbers less than one, and the product of the numerator and denominator of the fraction is 5040. S2 has not realized that the simple fractional form is also a condition of the positive rational number requested. When the Researcher asks the question, "Is a simple fractional form required for the positive rational number requested?". With this question, S2 realized that simple fractions also need to be considered in compiling the requested positive rational numbers. Next, S2 identifies the existing fractions one by one to determine the form of a simple fraction or not. The following are the results of the Researcher's interview with 'S2' in identifying the relevant reasons.

R : "Does $\frac{2}{2520}$ meet the requested rational number?"

S2 : "No, Sir, because $\frac{2}{2520}$ is not a simple fraction; if it is simplified, the result is no longer equal to 5040."

R : "Yes, that's right, then what is the difference between simple fractions and non-simple fractions?"

S2 : "If the form of a simple fraction cannot be simplified anymore, while the form of a non-simple fraction can still be simplified again."

R : "...If it can't be simplified anymore, then what is the common factor of the numerator and denominator?"

S2 : "...O yes, one, Sir!"

R "Yes, that's right, then what is the value of the greatest common factor of the numerator and denominator of non-simple fractions?"

S2 "S2 subject thinks and answers... Yes, more than one, Sir!"

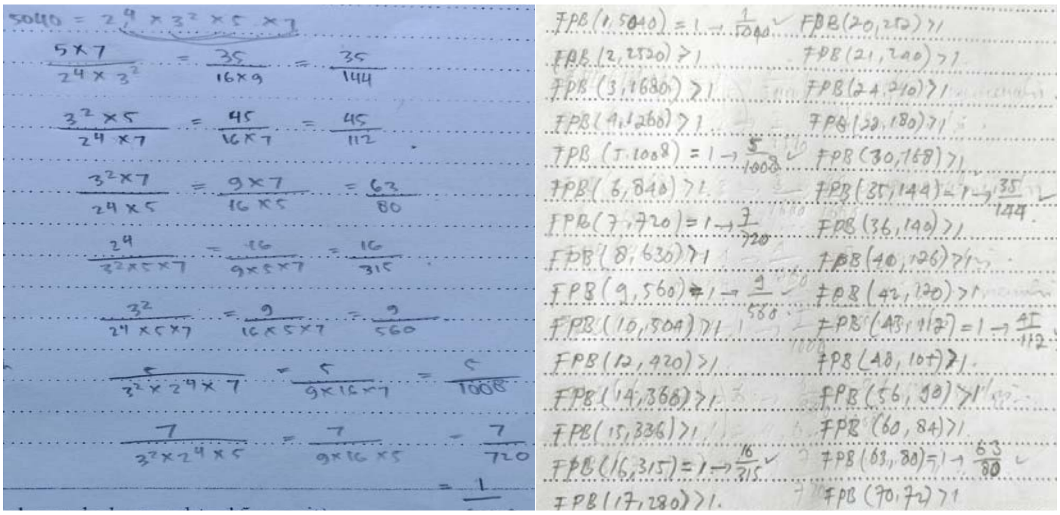
The data above show that the S2 subject's action situation identified the relevant reasons when determining the pattern of examples and non-examples of the requested rational numbers. Meanwhile, to realize the positive rational number needed to fulfill the GCD value equal to 1 must be done through a validation situation.

Meanwhile, S3 subjects to identify relevant reasons were only able to provide a few examples or not examples, such as $\frac{1}{5040}$ being an example being asked, while $\frac{2}{2520}$ was not an example being asked because it was not a simple fraction. When the Researcher asks "what is the difference between simple fractions and non-simple fractions?" S3 answered "simple fractions cannot be simplified anymore, while non-simple fractions can still be simplified". This shows that S3 has not been able to identify the relevant reasons.

Formulation Situation

The components of the critical thinking process related to situation formulation are evaluating relevant reasons and making inferences. The following is a description of the research results from the two components of the critical thinking process.

Evaluate relevant reasons. Both subjects can evaluate the relevant reasons under different conditions. Subject S1 can use a strategy by doing prime factorization from 5040 to 2^4, 3^2, 5, and 7. Furthermore, S1 can use numbers from prime factorization to arrange one-by-one positive rational numbers in less than one request. This strategy can be designed by S1 with critical reflection. While the subject of S2 can evaluate the relevant reasons by checking all fractions less than one that has been prepared previously using the concept of the GCD value equal to 1, using this strategy, S2 can show that all positive rational numbers less than 1 have a GCD value equal to 1, while for positive rational numbers more than one the GCD value is more than 1. This can be done by S2 with the help of scaffolding from researchers. Following are the responses given by subjects S1 and S2 in evaluating the relevant reasons, as shown in Figure 2.



Response S1 Response S2

Figure 2. Responses given by Subjects S1 and S2 in evaluating relevant reasons

Making inference. S1 and S2 subjects can consider conclusions based on their respective reasons. In contrast, S3 Subjects cannot consider conclusions for the right reasons. S1 considers conclusions based on reasoning from inductive results using prime factorization 5040. Based on inductive results, S1 can arrange positive rational numbers less than 1, which are asked for eight numbers. S1 says that "...that all the requested positive rational numbers are 16, but there are eight rational numbers that satisfy the requested condition...".

While S2 considers conclusions based on reasoning from inductive results using the results of a simple fraction examination using the concept of the GCD value equal to 1. Based on S2's inductive examination results, the number of positive rational numbers less than one request can be arranged as many as eight rational numbers. S2 says, " After checking that the positive rational number that satisfies the condition is 8."

The following are the considerations of the subject of S1 in making conclusions that are conveyed to the Researcher when the subject is interviewed by the Researcher as follows:

R : “Based on the information obtained, what conclusions can be drawn?”

S1 : “The number of positive rational numbers less than 1 with the product of the denominator and numerator equal to 5040 is eight numbers.”

R : "...What is the reason you can consider the conclusion that the positive rational number in question is 8?"

S1 : "Because by using a number from the prime factorization 5040, there are 16 positive rational numbers that can be made, but which fits the problem, there are only eight rational numbers."

The following are the results of the S2 subject interview data with researchers in considering the conclusions from solving the problems given as follows:

R : "What is the reason you can consider the conclusion that the positive rational number in question is 8?"

S2 : "Because based on the results of the examination using the GCD concept of 30 fractional numbers less than one that can be made, it turns out that there are eight simple fractional numbers that are less than 1."

The following are the results of the S3 subject interview data with researchers in considering the conclusions from solving the problems given as follows:

R : "What is the reason you can consider the conclusion that the positive rational number less than 1 in question is 3?"

S3 : "Because of the factorial number of 5040, which is 7x6x...x2x1, we get a number that satisfies the required rational number less than 1, namely $\frac{1}{5040}$, $\frac{5}{1008}$ and $\frac{7}{720}$. As for the numbers $\frac{2}{2540}$, $\frac{3}{1680}$, $\frac{4}{1260}$ and $\frac{6}{840}$ do not meet because they are non-simple fractions.

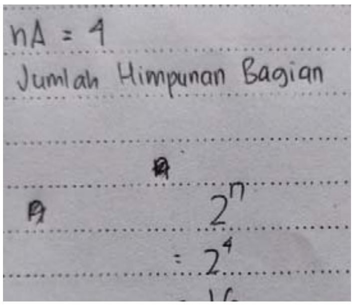
The data above shows that the subjects of S1 and S2 can logically consider their conclusions. S1 can consider conclusions with a unique argument (critical reflection), dan S2 can use explicit reflection. Sedangkan S3 considers conclusions with inappropriate reasons.

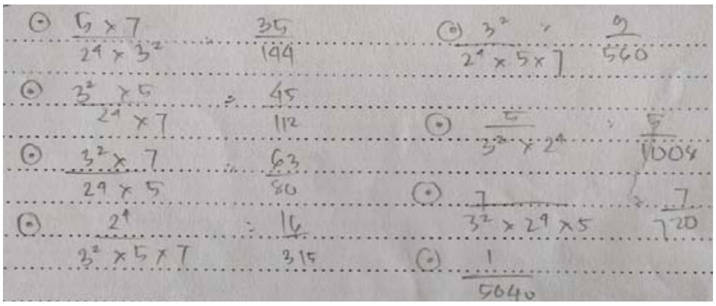
Validation Situation

The components of the critical thinking process related to situation validation are assessing the situation, providing clarity, and conducting an overview. The following is an explanation of the research results from the three components of the critical thinking process.

Assess the situation. S1 can assess the situation of conclusions by using deductive arguments, namely using the approach to the formula for the number of subsets that can be compiled from a set. This can be done by S1 with the help of scaffolding from the Researcher in the form of the question "if the prime factorization of 5040 is assumed to be a member of a set, written A = $\{2^4, 3^2, 5, 7\}$, can the formula for the number of subsets of A be used to calculate the number of rational numbers? With the help of this Scaffolding, S1 can use the formula for the number of subsets to assess the situation of the conclusions that have been considered. Based on the calculation results, S1 says that "Using the formula for the sum of subsets, we get 16 rational numbers, but because the positive rational number requested is a positive rational number less than 1, then the probability that fulfills half of 16 or there is 8".

While S2 can evaluate the conclusions that have been considered by using prime factorization 5040, this can be done by S2 from the Researcher's instructions and the results of group discussions with his friends. Based on the help of scaffolding, S2 can determine the factorization number 5040, which is 2^4,3^2,5,7. Then use number factorization to construct the necessary positive rational numbers. S2 says, based on his work, that "...by using prime factorization, 16 positive rational numbers can be constructed, but the sum of rational numbers in question is 8...". Following are the responses given by S1 and S2 in assessing the situation of the conclusions that have been considered.





The Response was given by S1 The Response was given by S2

Figure 3. The Response was given by S1 and S2 in considering alternative solutions

Based on the data above, it shows that S1 assesses the conclusion situation by considering deductive arguments, and this alternative solution can be done with the help of scaffolding from the Researcher. While S2 assesses the conclusion situation by considering inductive arguments and this alternative solution, S2 gets through group discussion activities by interacting with his friends in validation situations.

Provide clarity. Both subjects are required to explain the terms/language used in solving the problem. The validation situation through group discussions and presentations in front of the class is an opportunity for the subject to explain the terms used. The following is interview data when researchers and S1 in giving a description of some of the terms used in solving a given problem:

R : “Why is a simple fraction said to have a GCD value equal to 1?

S1 : " If the greatest common divisor of the numerator and denominator of a fraction is 1, then the fraction cannot be simplified any further."

R : “What is the relationship between GCD values and the concept of relatively prime?”

S1 : "If two numbers have a GCD value equal to 1, it means they are relatively prime. If the GCD value is more than 1, it means they are not relatively prime."

R : "What is prime factorization?"

S1 : "Prime factorization is a factor of a number that is a product of prime numbers."

R : “What is the relationship between the number of positive rational numbers in question and the subset?"

S1 : "The situation for the sum of subsets of a set is very similar to the sum of positive rational numbers in question, sir!"

The interview data above shows that S1 can explain the terms/concepts used in solving problems using their knowledge or actual development.

The following are the results of the Researcher's interview with 'S2' in providing clarity of problem-solving results through explicit reflection, as attached below:

R : “What is the relationship between positive rational numbers less than 1 and the value of GCD?”

S2 : “If a positive rational number is less than 1 then the GCD value of the numerator and denominator is 1"

R : “Why can you use the prime factorization of 5040 to determine a positive rational number less than 1?”

S2 : “Hmm... don't know sir!”

R : “At what time is the GCD of the two numbers equal to 1?”

S2 : "When both numbers are prime numbers... Sir!"

R : “How about GCD (4,3), is it more than 1?"

S2 : "Two numbers will have a GCD value equal to 1 if they are prime numbers or the product of prime numbers, sir!"

Based on the interview data above, it shows that S2 provides clarity of the terms used with the help of scaffolding types of hints from the Researcher.

Do an overview. The overview of this study is to check for errors in solving problems that have been done. Both subjects were allowed to examine the mistakes that might occur in solving problems in validation situations through group discussions and presentations in front of the class. One of the incorrect formulas used by S1 is the formula for the number of subsets of a set. Subject S1 gives a formula for the number of subsets of a set, i.e. n^2, where n is an integer. With the help of the 'S1 hint type scaffolding realized this error, and it can be corrected to 2^n. The following is the interaction data between the Researcher and the 'S1' in providing scaffolding during class presentations to identify errors in solving a given problem:

R : "Is it true that the formula for the number of subsets of a set is n^2?"
S1 : "Yes, sir, it seems to be correct because if we look at the results, there are 16 subsets!"
R : "…For example, if there is a set A = {1, 2, 3}, are there nine subsets that can be arranged?"
S1 : "[S1 tries to construct subsets of set A and says that...], there are eight subsets that can be arranged"
R : "Yeah, there's eight instead of 9... then where's the harm?"
S1 : "…Oh yes, Sir… now I remember, the correct formula is 2^n, not n^2
R : "That's right... so the formula has many subsets, which is 2^n, not n^2!"
S1 : "Yes, Sir, I remember now!"

The interaction data above shows that S1 can check the error of the formula given with the help of the type of scaffolding instructions from the Researcher in a validation situation.

Meanwhile, based on the results of S2 work and class presentations, it was revealed that there was an inaccuracy in the term used, namely using the term LCM from 5040 = 24.32.5.7. With the help of scaffolding, the S2 subject realized that an error had occurred and was able to make the correct correction, namely the prime factorization of 5040. The following is the Researcher's interaction data with S2' to identify errors in solving the given mathematical problems, as listed below:

R : "What does the LCM mean?"
S2 : "The lowest common multiple, Sir!"
R : "…Then what is the meaning of what is written here LCM of 5040 = $2^4.3^2.5.7$?"
S2 : "I mean, Sir, this 5040 is LCM of 2^4. 3^2. 5.7"
R : "Yes, but it is 24.32.5.7 times, which is a factor of 5040!"
S2 : "Yes, Sir, actually, I'm confused about what to name this!"
R : "Yes…, as you said that $2^4.3^2.5.7$ is a factor of 5040, right? So what factors can we say from the number?"
S2 : "If 2, 3, 5, 7 are prime numbers, Sir! But 2^4 and $3^{2,}$ the results are no longer prime numbers, so I'm confused, Sir!"

The interaction interview data above shows that S2 has not been able to check the errors of the terms used with the help of the type of scaffolding instructions from research in validation situations.

Discussion

The following is a summary of critical thinking processes S1, 'S2, and S3 in solving non-routine rational number problems, as shown in Table 5 below.

Table 5. Summary of Critical Thinking Process Data for Subjects S1, S2, And S3 in Solving Non-Routine Problems of Rational Numbers

Critical Thinking Process Components	Research subject		
	S1 (Critical Reflection)	**S2 (Explicit Reflection)**	**S3 (Not In Solving Problems)**
Identify focus	Can identify the focus of the problem well after being given scaffolding on the factorial concept	Can identify the focus of the problem with the help of scaffolding on the concept of rational numbers or/and factorial concepts	Can identify the focus of the problem with the help of scaffolding on the concepts of rational and factorial numbers
Identify and evaluate relevant reasons	- Can identify relevant reasons using his own knowledge; - Can evaluate relevant reasons by using prime factorization numbers in a unique way;	- Can identify relevant reasons by using scaffolding assistance; - Can evaluate the relevant reasons for using the GCD concept with the help of scaffolding (explicit reflection);	Unable to identify relevant reasons using own knowledge and scaffolding assistance;
Making inference	Can consider conclusions based on logical arguments uniquely (critical reflection).	Can consider conclusions based on logical arguments with the help of scaffolding (explicit reflection)	Consider conclusions based on imprecise/logical arguments.
Assess the situation	Can assess the situation of conclusions by using deductive rules with the help of scaffolding;	Can assess the situation of conclusions with alternative strategies using inductive arguments with the help of scaffolding;	Unable to judge the conclusion situation;
Provide clarity	Can provide clarity about the terms/concepts used by using their own knowledge	Can provide clarity of terms/concepts used by using scaffolding	Unable to provide clarity on the terms/concepts used
Do an overview	Can identify mistakes made with the help of scaffolding	He was unable to identify the error committed using neither his knowledge nor scaffolding's help.	Unable to identify error committed

Based on data analysis, it shows that the didactic situation that is built is very helpful for students in solving the given non-routine math problems and can think critically. This can be seen from the results in table 1 where 2 students (13.33%) can solve problems uniquely (critical reflection); 10 students (66.67%) who can solve problems with outside help (explicit reflection); and lastly there were only 3 students (20%) who could not solve the problem. This result is in accordance with the results of research (Danisman & Guler, 2019; Nickels & Cullen, 2017) which reported that Brousseau's didactic situation can increase learning activities and mathematical thinking.

The critical thinking process of subjects who experience critical reflection (S1) in solving non-routine rational number problems can be observed in the six components of the critical thinking checklist (Ennis, 1996b), namely identifying focus, identifying and evaluating relevant reasons, draw conclusions, assess the situation, provide clarity, and carry out an overview. This is in accordance with what was said (Ennis, 1991) that a person is said to have critical thinking at least there is a component with the acronym "FRISCO". The same thing is expressed (Facione, 2020) that a person will be able to think critically well if he meets the criteria of interpretation, analysis, evaluation, inference, explanation, and self-regulation.

S2 subjects who solve problems with explicit reflection can identify their critical thinking processes from identifying focus, identifying and evaluating reasons, making inferences, assessing the situation, and providing clarity. These results can be achieved with the help of scaffolding provided by researchers and the results of the subject's interaction with other students. This shows that S2 can solve a given problem by using potential development. This is in accordance with the results of research (Albert, 2021) that interactive critical thinking learning and providing scaffolding results in a significant increase in critical thinking for students. Wass et al. (2011) also said that verbal and conversational scaffolding allowed students to expand their ZPD to think critically. While the critical thinking process of S3 subjects who cannot solve problems can only identify the focus of the problem by using potential development while identifying and evaluating reasons can only provide examples and non-examples but cannot give good reasons. This shows that the existing knowledge on the subject is still very limited, so it cannot find the right relationship between what is known and what is being asked. This result is in accordance with the results of research (Rasiman, 2015) that students who are not critical can only identify facts that are given clearly, while identifying facts that are not clearly disclosed will experience obstacles.

Identifying focus is one component of a critical thinking checklist (Ennis, 1996b). In addition, identifying the focus of the problem is a sub-skill of critical thinking (Ennis, 1991), and is part of the interpretation skill (Facione, 1990; Watson & Glaser, 2002). Based on data analysis, information was obtained that the three subjects could identify the focus of the problem with the help of scaffolding by questioning unknown information. S1 subjects questioned the concept of 7 factorials, while S2 and S3 questioned the concept of 7 factorials as well as the concept of rational numbers. This shows that the three subjects can use potential development or do it through explicit reflection. This is in accordance with what was stated (Rønning, 2021) that the ideal adidactic environment provides feedback to students stating whether their responses are adequate with respect to the knowledge provided. S1 action situation in identifying the focus of the problem is to understand the concept of positive rational numbers while understanding the factorial concept turns into a validation situation. Meanwhile, S2 and S3 action situations to identify the focus of the problem must turn into a validation situation because he cannot independently understand the concept of rational numbers and factorial concepts. This finding suggests that both subjects can identify the focus of the problem by using potential development or achieving explicit reflection. This is also in line with the results of research (Basri et al., 2019) that students' interpretation skills are still low. This is also in line with the results of research (Van Der Stuyf, 2002) that scaffolding can facilitate students' ability to build prior knowledge and internalize new information.

Identifying reasons is one component of the critical thinking checklist (Ennis, 1996b). Hitchcock (2017) lists the components of the critical thinking process related to gathering evidence. Identifying reasons is a critical thinking sub-skill of analyzing skills (Facione, 1990; Halpern, 2012). Based on data analysis, S1 and S2 can identify relevant reasons by identifying possible answers/solutions from several examples and non-examples that can be given. This can be done by S1 based on their knowledge, actual development, or implicit reflection. While Masters can identify reasons with the help of scaffolding, this shows that Masters can do it by using potential development or through explicit reflection. This is in accordance with the results of research by (Rokhmat et al., 2019) that with scaffolding, students are able to solve problems that exceed their abilities. Based on the didactic situation, S1 can identify the relevant reasons in the action situation. In contrast, the

S2 subject can identify the relevant reasons for the validation situation. Meanwhile, S3 cannot identify the relevant reasons, both using actual development and potential development. This is in accordance with the results of research (Basri et al., 2019) that students' analytical skills are still not good.

Evaluating reason is one component of the critical thinking checklist (Ennis, 1996b). Hitchcock (2017) lists the components of the critical thinking process related to assessing evidence. Evaluation of reasons is part of the critical thinking skills of evaluation (Facione, 1990; Watson & Glaser, 2002). Evaluation means differentiating values using certain criteria and making comparisons (Omar et al., 2012). Based on data analysis, S1 can evaluate the relevant reasons by prime factorizing the number 5040 and using that number to identify the rational number in question. This can be done uniquely through critical reflection. In contrast, S2 evaluates the relevant reasons by calculating the GCD value to find a simple fraction form of a fraction whose product of the denominator and numerator is equal to 5040. This can be done by S2 using actual development or through implicit reflection. This can happen because, as research results (Van Der Stuyf, 2002), scaffolding can facilitate students' ability to build on previous knowledge and internalize new information.

Making inferences is a component of the critical thinking checklist (Ennis, 1996b). Hitchcock (2017) lists these components of the critical thinking process by assessing inference. Inference is also a critical thinking skill (Ennis, 2015; Facione, 2020; Watson & Glaser, 2002). Inference means identifying and securing the elements needed to draw reasonable conclusions (Facione, 2011). Setiana et al. (2021) say that inference requires an explanation of the given findings. Based on data analysis, 'S1 and S2 can consider conclusions with logical reasons. S1 can consider findings by using the induction of the prime factorization of the number 5040. Meanwhile, 'S2 can consider conclusions based on the results of inductive arguments using the concept of GCD values to test simple forms. This can be done by S1 using the results of critical reflection, while S2 uses the results from the actual development. This result is certainly not to the results of research (Aiyub et al., 2021; Basri et al., 2019; Seventika et al., 2018), which concluded that students' inference abilities were low. The scaffolding assistance received and subject interaction in didactic situations are thought to have helped provide good inference. In addition, evaluating relevant reasons and making inferences is a formulation situation for S1 and S2. It is appropriate (Suryadi, 2019b) to say that when a new mental object begins to form, Brousseau describes it as a formulation situation.

Assessing situations is a critical thinking checklist (Ennis, 1996b). An appropriate component of the critical thinking process (Hitchcock, 2017) is considering other relevant information. Based on data analysis, to assess the situation of the conclusions drawn, each subject can evaluate the status of the findings under different conditions. S1 assesses the conclusion situation by using deductive arguments, namely by using the formula for many subsets of the set. The results using this formula support the conclusions that have been considered. This can be done in S1 by using potential development or achieving explicit reflection. In contrast, the S2 subject can assess the conclusion situation by using inductive arguments, namely by determining the prime factorization of the number 5040 and using that number to construct the positive rational number in question. The results obtained by S2 using this strategy also support the conclusions that have been considered. This can be done 'S2 by using potential development or achieving explicit reflection. This is in accordance with the research results of (Wass et al., 2011) that verbal and conversational scaffolding allows students to expand their ZPD to think critically.

Providing clarity is one component of the critical thinking checklist (Ennis, 1996b). The indicator of clarity in this research is that it can explain the conclusions given. The description of the alternative results of the critical thinking process (Hitchcock, 2017) considers other relevant information. Giving an answer is one of the subcritical thinking skills (Facione, 1990a). Experts define explaining as to the ability to present cognitively and consistently the results of one's reasoning (Facione, 2020). Based on data analysis, S1 can explain the terms GCD, relatively prime, prime factorization, and subsets used using actual development. While S2 can define the term GCD with actual development, the terms relative prime and prime factorization can be defined using potential development. This shows that S1 can provide clarity by using implicit reflection and S2 by using explicit reflection. Basri et al. (2019) found that the cause of students failing to make good explanations was that students' knowledge of mathematical concepts was still low. Hamel et al. (2015) found that an increase in explaining skills coincided with an increase in students' knowledge. This shows that assessing the situation and providing explanations becomes a validation situation for S1 and S2. It is appropriate to say (Suryadi, 2019b) that the process of interaction between students and between teachers and students allows the negotiation of meaning so that arguments, claims, or representations develop that encourage both internal and external validation processes.

Conducting an overview is a component of a critical thinking checklist (Ennis, 1996b). Hitchcock (2017) lists this last component of the critical thinking process as an overall assessment. Doing an overview is an essential part of thinking skills from self-regulation (Facione, 2020). Setiana et al. (2021) carried out an overview of the work results to know whether the problem solving was correct or if there was something that needed to be fixed. Based on data analysis for the overview, S1 can check for errors in the formula for the number of subsets used by using potential development. Meanwhile, S2 cannot check for errors in using the term LCM for prime factorization of 5040 using both actual and potential development. The ability to do a review is part of the ability to self-regulation. Skepticism can encourage a person to reflect so as to produce the right conclusions and make the right decisions (Nugroho et al., 2018). This shows that doing an overview becomes a validation situation for S1 and an institutional situation for S2.

Conclusion

Based on data analysis and discussion, the student's critical thinking process in solving problems in a unique way (critical reflection) can be identified by the six components of critical thinking from Ennis, namely identifying focus, identifying reasons and evaluating relevant reasons, making inferences, assessing the situation, providing clarity, and conducting an overview. Meanwhile, students' critical thinking processes in solving problems with the help of others (explicit reflection) can be identified as five of the six components of critical thinking from Ennis, namely identifying focus, identifying reasons and evaluating relevant reasons, making inferences, assessing situations, and providing clarity. The critical thinking process of students who cannot solve problems, both implicit and explicit reflection, can only be identified as two of the six critical thinking components of Ennis, namely identifying a focus and making inferences in solving problems.

Another finding is the transfer of responsibility for didactic situations from teachers to students towards subjects who can solve problems critically reflection higher than students who can solve problems with explicit reflection.

Acknowledgments

My gratitude goes to the Ministry of Religion of the Republic of Indonesia, and this paper is partially supported by scholarships from 5000 Doctoral Programs.

References

Aiyub, Suryadi, D., Fatimah, S., & Kusnandi. (2021). Investigation of Watson-Glaser critical thinking skills of junior high school students in solving mathematical problems. *Journal of Physics: Conference Series*, *1806*(1), 012090. https://doi.org/10.1088/1742-

6596/1806/1/012090
Akgun, A., & Duruk, U. (2016). The Investigation of Preservice Science Teachers' Critical Thinking Dispositions in the Context of Personal and Social Factors. *Science Education International*, *27*(1), 3–15.
Albert, S. (2021). Applied critical thinking in strategy: Tools and a simulation using a live case. *Journal of Education for Business*, *96*(4), 252–259. https://doi.org/10.1080/08832323.2020.1792395
Ansari, B. I., Saleh, M., Nurhaidah, & Taufiq. (2021). Exploring students' learning strategies and self-regulated learning in solving mathematical higher-order thinking problems. *European Journal of Educational Research*, *10*(2), 743–756. https://doi.org/10.12973/eu-jer.10.2.743
Artigue, M., & Blomhøj, M. (2013). Conceptualizing inquiry-based education in mathematics. *ZDM - International Journal on Mathematics Education*, *45*(6), 797–810. https://doi.org/10.1007/s11858-013-0506-6
As'ari, A. R., Mahmudi, A., & Nuerlaelah, E. (2017). Our prospective mathematic teachers are not critical thinkers yet. *Journal on Mathematics Education*, *8*(2). https://doi.org/http://dx.doi.org/10.22342/jme.8.2.3961.145-156
Basri, H., Purwanto, As'ari, A. R., & Sisworo. (2019). Investigating critical thinking skill of junior high school in solving mathematical problem. *International Journal of Instruction*, *12*(3), 745–758. https://doi.org/10.29333/iji.2019.12345a
Beyer, B. . (1985). Critical thinking: what is it? *Social Education*, *49*(4), 270-276.
Biber, A. C., Tuna, A., & Incikabi, L. (2013). An investigation of critical thinking dispositions of mathematics teacher candidates. *Educational Research*, *4*(2), 109–117.
Brousseau, G. (2002). Theory of Didactical Situations in Mathematics. In R. S. and V. W. Nicola Balacheff, Mantin Cooper (Ed.), *Kluwer Academic Publishers* (Edited and). Kluwer Academic Publishers. https://doi.org/10.1007/0-306-47211-2
Changwong, K., Sukkamart, A., & Sisan, B. (2018). Critical thinking skill development: Analysis of a new learning management model for Thai high schools. *Journal of International Studies*, *11*(2), 37–48.
Choy, S. C., & Oo, P. S. (2012). Reflective Thinking and Teaching Practices: a Precursor for Incorporating Critical Thinking Into the Classroom? *International Journal of Instruction*, *5*(1), 167–182.
Chusni, M. M., Saputro, S., Rahardjo, S. B., & Suranto. (2020). Student's Critical Thinking Skills Through Discovery Learning Model Using E-Learning on Environmental Change Subject Matter. *European Journal of Educational Research*, *9*(3), 1257–1265. https://doi.org/10.12973/eu-jer.10.3.1123
Creswell, J. W. (2007). *Qualitative Inquiry & Research Design: Choosing Among Five Approaches* (Second Edi). Sage Publication, Inc.
Danisman, S., & Guler, M. (2019). A problem-solving process using the Theory of Didactical Situations: 500 lockers problem. *Inovacije u Nastavi*, *32*(1), 105–116. https://doi.org/10.5937/inovacije1901105d
Dewey, J. (1910). How We Think. In *D. C. Heath & Co*. https://doi.org/10.2307/1415632
Dewey, J. (1933). How We Think: A Restatement of the Relation of Reflective Thinking to the Educative Process. Boston, MA: D.C. Heath & Co Publishers. In *D.C. Heath and Company*.
Dubinsky, E. (2001). Using a Theory of Learning in College Mathematics Courses. *MSOR Connections*, *1*(2), 10–15. https://doi.org/10.11120/msor.2001.01020010
Eatough, V., & Smith, J. (2017). *Interpretative Phenomenological Analysis. In: Willig, C. and*

Stainton-Rogers, W. (eds.) Handbook of Qualitative Psychology. Sage Publication Ltd.

Ennis, R. H. (1991). Critical Thinking: A Streamlined Conception. *Teaching Philosophy*, *14*(1), 5–24.

Ennis, R. H. (1996a). Critical thinking. In *Upper Saddle River: Prentice-Hall.*

Ennis, R. H. (1996b). Critical Thinking Dispositions: Their Nature and Assessability. *Informal Logic*, 165–182.

Ennis, R. H. (2015). The Nature of Critical Thinking: Outlines of General Critical Thinking Disposition and Abilities. *Sixth International Conference on Thinking at MIT*, 1–8. https://education.illinois.edu/docs/default-source/faculty-documents/robert-ennis/thenatureofcriticalthinking

Ersoy, E., & Başer, N. (2014). The Effects of Problem-based Learning Method in Higher Education on Creative Thinking. *Procedia - Social and Behavioral Sciences*, *116*(February 2014), 3494–3498. https://doi.org/10.1016/j.sbspro.2014.01.790

Facione, P. A. (1990a). Critical Thinking : A Statement of Expert Consensus for Purposes of Educational Assessment and Instruction Executive Summary " The Delphi Report. *The California Academic Press*, *423*(c), 1–19. http://www.insightassessment.com/pdf_files/DEXadobe.PDF

Facione, P. A. (1990b). Critical Thinking: A Statement of Expert Consensus for Purposes of Educational Assessment and Instruction. Research Findings and Recommendations. *American Philosophical Association.*

Facione, P. A. (2011). Critical Thinking : What It Is and Why It Counts. *Insight Assessment*, *ISBN 13: 978-1-891557-07-1.*, 1–28. https://www.insightassessment.com/CT-Resources/Teaching-For-and-About-Critical-Thinking/Critical-Thinking-What-It-Is-and-Why-It-Counts/Critical-Thinking-What-It-Is-and-Why-It-Counts-PDF

Facione, P. A. (2015). Critical Thinking : What It Is and Why It Counts. In *Insight assessment* (Issue ISBN 13: 978-1-891557-07-1.). https://www.insightassessment.com/

Facione, P. A. (2018). Critical Thinking: Process , Purpose , or Outcome ? *Measured Reasons LLC*, 1–2. www.measuredreasons.com

Facione, P. A. (2020). Critical Thinking: What It Is and Why It Counts. *Insight Development Program*, *53*(9), 1–31. https://doi.org/10.1017/CBO9781107415324.004

Fadhilah, R., Sujadi, I., & Siswanto. (2021). The Critical Thinking Process of Senior High School Students in Problem-Solving of Linear Equations System. *IOP Conference Series: Earth and Environmental Science*, *1808*(1), 1–7. https://doi.org/10.1088/1742-6596/1808/1/012063

Fahy, P. J. (2002). Assessing critical thinking processes in a computer conference. In *Centre for Distance Education Athabasca University Retrieved August* (Vol. 15). http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.169.9039&rep=rep1&type=pdf

Fisher, A. (2001). *Critical Thinking. An Introduction* (Frist Publ). Cambridge University Press. https://doi.org/10.2307/2019787

Halpern, D. F. (2012). *Halpern Critical Thinking Assessment [Measurement instrument]. Mödling: Schuhfried*.

Harel, G. (2008). *What is Mathematics? A Pedagogical Answer to a Philosophical Question*. https://doi.org/10.5948/upo9781614445050.018

Harel, G., & Sowder, L. (2005). Advanced Mathematical- Thinking at Any Age: Its Nature and Its Development. *Mathematical Thinking and Learning*, *7*(1), 27–50. https://doi.org/10.1207/s15327833mtl0701_3

Hartman, H. (2002). Scaffolding & Cooperative Learning. In *Human Learning and Instruction*. City

College of City University of New York.

Henningsen, M., & Stein, M. K. (1997). Mathematical tasks and student cognition: Classroom-based factors that support and inhibit high-level mathematical thinking and reasoning. *Journal for Research in Mathematics Education*, *28*(5), 524–549.

Hitchcock, D. (2017). Critical Thinking as an Educational Ideal. In *On Reasoning and Argument, Argumentation Library* (pp. 477–497). © Springer International Publishing. https://doi.org/10.1007/978-3-319-53562-3_30

Hitchcock, D. (2018). Critical Thinking. In *Stanford Encyclopedia of Philosophy* (First publ). Stanford Encyclopedia of Philosophy.

Jenicek, M., & Hitchcock, D. (2005). *Evidence-based practice: Logic and critical thinking in medicine. Chicago*. AMA Press.

Kardoyo, Nurkhin, A., Muhsin, & Pramusinto, H. (2020). Problem-based learning strategy: Its impact on students' critical and creative thinking skills. *European Journal of Educational Research*, *9*(3), 1141–1150. https://doi.org/10.12973/EU-JER.9.3.1141

Kriswantoro, Kartowagiran, B., & Rohaeti, E. (2021). A critical thinking assessment model integrated with science process skills on chemistry for senior high school. *European Journal of Educational Research*, *10*(1), 285–298. https://doi.org/10.12973/EU-JER.10.1.285

Krulik, S., & Rudnick, J. A. (1988). *Problem Solving: A Handbook for Elemen School Teachers*. Allyn and Bacon, Inc.

Lai, E. R. (2011). Critical Thinking: A Literature Review. Research Report. *Pearson*, *35*(3), 219–225. https://doi.org/10.1046/j.1537-2995.1995.35395184278.x

Marcia, D. (1991). Group Discussion and Individual Critical Thinking Processes: An Interactive Perspective. *Paper Presented at the Annual Meeting of the Central States Communication Association*, 1–24. https://files.eric.ed.gov/fulltext/ED336772.pdf

Mason, M. (2007). Critical thinking and learning. *Educational Philosophy and Theory*, *39*(4), 339–349. https://doi.org/10.1111/j.1469-5812.2007.00343.x

Mullis, I. V. S., Martin, M. O., Foy, P., & Arora, A. (2015). Timss 2015 International Results in Mathematics. *TIMSS & PIRLS*. http://timss2015.org/timss-2015/science/student-achievement/distribution-of-science-achievement/

National Council of Teachers of Mathematics (NCTM). (2000). Principles and Standards for School Mathematics. United States of America. *The National Council of Teachers of Mathematics, Inc.*

Nickels, M., & Cullen, C. J. (2017). Mathematical thinking and learning through robotics play for children with critical illness: The case of Amelia. In *Journal for Research in Mathematics Education* (Vol. 48, Issue 1). https://doi.org/10.5951/jresematheduc.48.1.0022

Nugroho, P. B., Nusantara, T., As'ari, A. R., Sisworo, Hidayanto, E., & Susiswo. (2018). Critical thinking disposition: Students skeptic in dealing with ill-logical mathematics problem. *International Journal of Instruction*, *11*(3), 635–648. https://doi.org/10.12973/iji.2018.11343a

OECD. (2019). PISA 2018 Results. Combined Executive Summaries. *Journal of Chemical Information and Modeling*, *53*(9), 1689–1699.

Omar, N., Haris, S. S., Hassan, R., Arshad, H., Rahmat, M., Zainal, N. F. A., & Zulkifli, R. (2012). Automated Analysis of Exam Questions According to Bloom's Taxonomy. *Procedia - Social and Behavioral Sciences*, *59*(October), 297–303. https://doi.org/10.1016/j.sbspro.2012.09.278

Pfister, M., Moser Opitz, E., & Pauli, C. (2015). Scaffolding for mathematics teaching in inclusive primary classrooms: a video study. *ZDM - Mathematics Education*, *47*(7), 1079–1092. https://doi.org/10.1007/s11858-015-0713-4

Rasiman. (2013). *Proses Berpikir Kritis Siswa dalam Menyelesaikan Masalah Matematika Ditinjau dari Perbedaan Kemampuan Matematika*. Unesa.

Rasiman. (2015). Leveling of Critical Thinking Abilities of Students of Mathematics Education in Mathematical Problem. *IndoMS-JME*, *6*(1), 40–52. https://doi.org/dx.doi.org/10.22342/jme.61.40

Ricoeur, P. (1986). *Lectures on Ideology and Utopia*. New York: Columbia University Press. https://archive.org/details/pdfy-oRPzWEh3nXrYxehT/page/n13/mode/2up

Rokhmat, J., Marzuki, Wahyudi, & Putrie, S. D. (2019). A strategy of scaffolding development to increase students' problem-solving abilities: The case of physics learning with causalitic-thinking approach. *Journal of Turkish Science Education*, *16*(4), 569–579. https://doi.org/10.36681/tused.2020.8

Rønning, F. (2021). Opportunities for language enhancement in a learning environment designed on the basis of the theory of didactical situations. *ZDM - Mathematics Education*, *53*(2), 305–316. https://doi.org/10.1007/s11858-020-01199-x

Rudi, R., Suryadi, D., & Rosjanuardi, R. (2020). Teachers' Perception as a Crucial Component in the Design of Didactical Design Research-Based Teacher Professional Learning Community in Indonesia. *European Online Journal of Natural and Social Sciences*, *9*(3), 642–654. http://www.european-science.com

Setiana, D. S., Purwoko, R. Y., & Sugiman. (2021). The application of mathematics learning model to stimulate mathematical critical thinking skills of senior high school students. *European Journal of Educational Research*, *10*(1), 509–523. https://doi.org/10.12973/EU-JER.10.1.509

Seventika, S. Y., Sukestiyarno, Y. L., & Mariani, S. (2018). Critical thinking analysis based on Facione (2015) - Angelo (1995) logical mathematics material of vocational high school (VHS). *Journal of Physics: Conference Series*, *983*(1). https://doi.org/10.1088/1742-6596/983/1/012067

Strømskag, H. (2017). A methodology for instructional design in mathematics—with the generic and epistemic student at the centre. *ZDM - Mathematics Education*, *49*(6), 909–921. https://doi.org/10.1007/s11858-017-0882-4

Suryadi, D. (2011). Metapedadidaktik dalam Pembelajaran Matematika : Suatu Strategi Pengembangan Diri Menuju Guru Matematika Profesional [Metapedadidactics in Mathematics Learning: A Self-Development Strategy Towards Professional Mathematics Teachers]. Pidato Pengukuhan Guru. In *Tidak Diterbitkan*.

Suryadi, D. (2019a). *Landasan Filosofis Penelitian Desain Didaktis (DDR) [Philosophical Foundations of Didactic Design Research (DDR)]*. (T. G. Press (ed.); Cetakan 1). Gapura Press.

Suryadi, D. (2019b). *Penelitian Desain Didaktical (DDR) dan Implementasinya* (A. S. Maulida (ed.); Cetakan 1). Gapura Press.

Tall, D. (1999). The cognitive development of proof: Is mathematical proof for all or for some? *Developments in School Mathematics Education Around the World*, *4*(August), 117–136.

Tohir, M., Maswar, M., Mukhlis, M., Sardjono, W., & Selviyanti, E. (2021). Prospective teacher's expectation of students' critical thinking process in solving mathematical problems based on Facione stages. *Journal of Physics: Conference Series*, *1832*(1). https://doi.org/10.1088/1742-6596/1832/1/012043

Van de Pol, J., Volman, M., & Beishuizen, J. (2010). Scaffolding in teacher-student interaction: A decade of research. *Educational Psychology Review*, *22*(3), 271–296. https://doi.org/10.1007/s10648-010-9127-6

Van Der Stuyf, R. R. (2002). Scaffolding as a Teaching Strategy – Definition and Description. In *Adolescent Learning and Development* (pp. 1–13).

Wass, R., Harland, T., & Mercer, A. (2011). Scaffolding critical thinking in the zone of proximal development. *Higher Education Research and Development*, *30*(3), 317–328. https://doi.org/10.1080/07294360.2010.489237

Watson, G., & Glaser, E. (2002). Watson Glaser Critical Thinking Appraisal Test. In *Pearson Assessment*.

Weiss, C. A. (1993). But how do we get them to think? In *M.Svinicki (Ed), Teaching excellence: Toward the best in the academy*. Ames, IA: PODNetwork.

Wood, D., Bruner, J. S., & Ross, G. (1976). the Role of Tutoring in Problem Solving. *Journal of Child Psychology and Psychiatry*, *17*(2), 89–100.

Zaretskii, V. K. (2009). The Zone of Proximal Development. *Journal of Russian & East European Psychology*, *47*(6), 70–93. https://doi.org/10.2753/rpo1061-0405470604