Forecasting volatility and Value-at-Risk of the Karachi Stock Exchange 100 index: Comparing distribution-type and asymmetry-type models

Abdul Haque, Kashif Naeem

Department of Management Science, COMSATS Institute of Information Technology, Lahore, Pakistan

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Abstract

We investigate the daily volatility and Value-at-Risk (VaR) forecasts for the Karachi Stock Exchange 100 Index (KSE-100) series from 1998 to 2008. The forecasting performance of the distribution-type volatility models (GARCH-N, -t, -SGT, and -HT) are compared with that of asymmetry-type models (GJR-GARCH and EGARCH) in order to ascertain the crucial determinants for improving forecast accuracy of daily volatility and VaR. Empirical results indicate that the GARCH-HT and GARCH-SGT models generate far more accurate daily volatility forecasts as compared to their competitors. For VaR calculation, the GARCH-t and GARCH-SGT are the appropriate models to predict the daily VaRs of KSE-100 stock index under high confidence level.

Keywords: GARCH; Returns distribution; Volatility asymmetry; Daily price range; KSE

Introduction

Financial crisis, caused by high price movements, irrational behavior of investors and ending up with financial debris of bankruptcy, bailout packages or even the complete extinction from the market have always been fatal. In addition, not only the national and institutional losses are observed but one who is hit the hardest is the individual investor. The developed countries, despite of strong financial markets, sound baking system and well informed investor experienced huge financial losses and further ensuing to a deteriorating national and international investment. (Wall street crisis of 1987).

The modeling and forecasting of the volatility has ever been an eminent feature of finance literature, different techniques and models have been developed to meet the growing requirements. Often the volatility in return's series exhibit different patterns such as mean reversion, persistence, clustering etc also their distribution is found to be non-normal (leptokurtic) skewness and kurtosis and these multi-faceted characteristics make the modeling and forecasting even more challenging. While studying and forecasting the volatility of financial series a huge portion of literature rely on the generation of generalized conditional heteroskedasticity (GARCH and ARCH) models, developed by Engle (1982) and Bollerslev (1986). The GARCH generations of models not only explore the basic auto-regressive structure of conditional variance but also have the flexibility to accommodate for numerous nuisance characteristics of returns volatility.

The observed financial volatility not only necessitates for accurate out of sample forecast but also demands the advanced and sophisticated models to forecast the volatility in a risk management perspective. Value at Risk (VaR) is the most widely used technique for market risk estima-

Corresponding author: Abdul Haque, Department of Management Science, COMSATS Institute of Information Technology, Lahore, Pakistan, E-mail: ahaque@ciitlahore.edu.pk

tions. VaR enables the financial analyst and risk managers to forecast/calculate the worst/highest a portfolio's possible risk/loss for a selected period of time and at a given level of significance. The available GARCH genre of models for modeling and forecasting the volatility can be classified into distribution type and asymmetric type GARCH models. The distribution type GARCH models are the classical ARCH/GARCH models, which allow for a number of alternative distributional assumptions. The flexibility to permit a number of alternative error distributions enables to calculate and compare the forecast ability of these models for a range of financial markets. Numerous studies have been conducted on the financial markets of the developed countries and the reported results have been found improving and varying for the stated alternative error distributions Wilhelmsson (2006) and Chuang *et al*. (2007). By applying the symmetric GARCH (1,1) with alternative error distributions on S&P 100 index future returns, Wilhelmsson (2006) reported that predictive ability of the GARCH (1,1) with leptokurtic error distributions was significantly improved in comparisons to other alternatives, allowing for the kurtosis (skewness) did improve the forecasts significantly. Nevertheless, use of a complex distribution in place of standard normal distribution does not guarantee improving forecasts. Chuang et .al (2007) analyzed the forecasting performance of linear GARCH model with various distributional assumptions by using the data of stock market and exchange rate. The estimated results show that a simple distribution may perform no worse than a complex one.

The asymmetric type GARCH models use flexible volatility specifications to accommodate for real life asymmetries and then examining the forecasting abilities. These models provide a useful way to overcome the inabilities of classical GARCH models. Among the researchers who advocate asymmetric type GARCH models to predict stock returns are Franses and van Dijk (1996) and Wei (2002) propose the Quadratic-GARCH model while Brailsford and Faff (1996) and Taylor (2004) report the results in favor of GJR-GARCH model. Whereas, the studies which report the superior predictive performance of EGARCH model include Heynen and Kat (1994), Chong *et al* (1999) and Loudon *et al* (2000). Briefly, the above mentioned studies support the significant role of asymmetries in volatility forecast. In the case of emerging equity markets, Gokcan (2000) reported that simple GARCH (1, 1) model performs significantly better than EGARCH model, no matter even if the returns series are skewed as well. While forecasting the stock market volatility through asymmetric type EGACRH and the simple GARCH model, McMillan *et al* (2000) found that earlier does not outperform the later. Ng and McAleer study the forecasting ability of GARCH(1,1) and GJR-GARCH(1,1) type models and Risk Metrices model by using the stock market volatility and report the GARCH type models to superior over the later.

Although, existing literature on modeling and forecasting the stock market volatility provides us with a wide range of models, yet the selection of a model on the optimum performance criterion is an astute one. Besides, toward the increasing accuracy of models, the literature is missing the potential contribution of distributional assumptions and volatility specifications. An empirical study exploring the significant contribution of these ignored sources of information would be interesting, if any?

While evaluating the forecasting performance of different GARCH models, choosing a surrogate of true daily volatility is a practical challenge. A great number of studies have proxied the ex post latent volatility with squared daily returns Brooks & Persand (2002), Awartani and Corradi (2005) and Sadorsky (2006). Using a squared daily return as a proxy is likely to under estimate the forecasting performance of GARCH models since it suffers from daily market noise and may not be a true estimate of variance. Because of the practical difficulty to proximate the volatility, some studies advocate that inappropriate volatility proxy (squared return) cause the GARCH models to generate inaccurate forecasts (Anderson and Bollerslev (1997, 1998) and McMillan and Speight (2004)). An alternative suggested to avoid the incorrect conclusions is to use the high-frequency data (intraday) to proxy the true daily volatility. A sum of squared intraday return, obtained from intraday data may well proxy the daily volatility. The earlier studies using GARCH gene of models to forecast the volatility did not pay much attention to the distributional assumptions and asymmetries, a potential source of improvements in the forecasts. As an empirical effort, this study fills gap in literature by investigating the forecasting performance of distribution-type (GARCH-N,T,HT and SGT) and asymmetry-type (GJR,E-GARCH) GARCH models.

This study is an empirical effort on Karachi Stock Exchange (KSE-100) index series over the sample period, 1 January 1998 to 30 September 2008, consisting of 2,613 daily observations. The study focuses on the volatility/VaR forecasting performance between distribution-type and asymmetric-type GARCH models using a rolling-window technique. The sample size for each rolling-window was set to 1000 observations over the last 493 daily observations. Four different types of loss functions, namely MAE, MSE, MME(O) and MME(U) respectively have been calculated to compare the forecasting performance of the aforementioned types of models.

Equity Market in Pakistan

Currently there are three stock exchanges in Pakistan, namely Karachi, Lahore and Islamabad stock exchange. Among these three the Karachi Stock Exchange (KSE) is the largest, oldest (since 1947) and internationally recognized. The study focuses on the KSE only and utilizes the KSE-100 index for the empirical estimation and analysis. The KSE-100 was introduced with 1000 base points in 1991 and includes those 100 companies which cover almost 80% of market capitalization. The KSE observed a tremendous growth in terms of index, market capitalization and the number of companies listed. The government's liberalization and openness policy provided a further impetus to this ongoing growth and attracted a good number of foreign investors. The market not only grew locally but was also acknowledged globally as in 2002 the KSE was announced as the best performing market in the world by "Business Week". This sound and fundamental supported growth provided the ground for the derivatives/futures trading and the government launched Pakistan's National Commodity Exchange Limited (NCEL) and introduced the limited/selected commodities futures contracts in 2003.

By the October, 2004 KSE index reached up to the level of 5245.82 with the market capitalization of US $ 25.23 billion. Year 2004 was the successive 3rd year of best performance as the "US Today" also acknowledged the KSE as one of the best performing bourses in the world. The market also grew in terms of number of companies listed as by the year 2004, there were 663 companies with a paid up capital of US $ 6.59 billion. The market was ranked 1st and 3rd in terms of turnover ratio in the year 2003 and 2006 respectively (Global Stock Markets Fact book, 2004, 2007). The years 2006 and 2007 proved to be very frugal as the foreign buying kept on increasing and according to the State Bank of Pakistan (SBP) it reached up to US $ 523 in 2007. By the end of year 2007, the 754 companies had been listed with the market capitalization of US $ 52 billion and a paid up capital of US $ 8.27 billion. In the year 2008 the index peaked 15,737.32 with the market capitalization of US $ 23490665415.48. An increase of 7.4% in KSE-100 in the year 2008 once again made KSE the best performer among the emerging markets (Gulf News).

Despite the strong growth and development of KSE, suddenly in the index started shrinking and dropped to the level of 5000 in year 2009, just in a period of year. This dramatic fall in the index put both of the policy makers and investors to calculate the possible fall and financial loss. Current study is an effort to provide the critical information to the investor in addition to a help to the policy makers.

Data and methodology

Data

The data illustrated in this study are daily price data of the Karachi Stock Exchange 100 Index (KSE-100 Index) obtained from the website of www.brecoder.com. The sample period spans from 1 January 1998 to 30 September 2008 for a total of 2613 trading days, and includes high, low and closing prices[1]. The sample is divided into two parts, as shown in Fig. 1. The first 1196 observations (1 January 1998 to 31 December 2002) are used as the in-sample for estimation, while the remaining 1417 observations (1 January 2003 to 30 September 2008) are taken as the out-of-sample for forecast evaluation.

[1] The daily high and low prices data is used to calculate the daily volatility proxy in our evaluation of volatility forecasts performance.

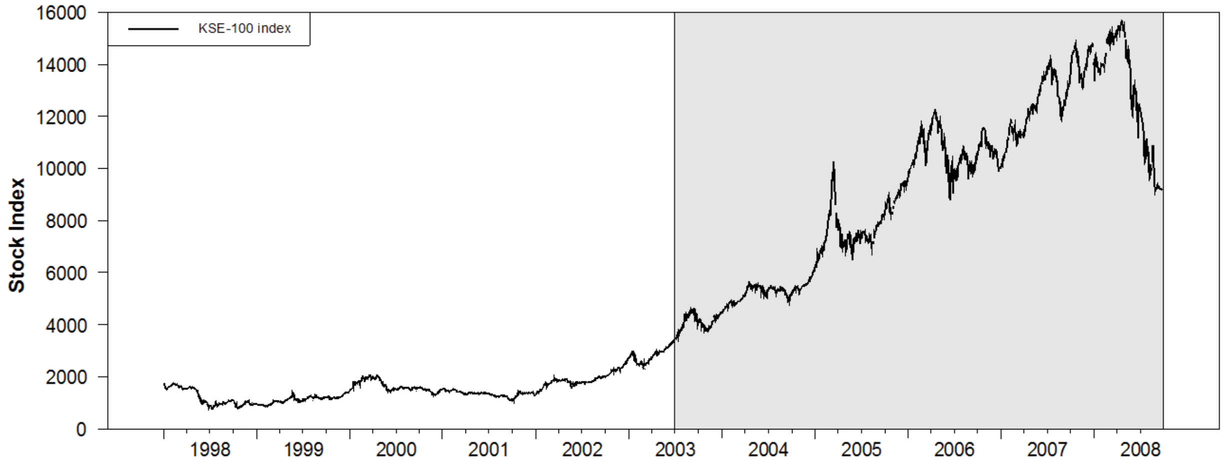

Figure. 1. Daily KSE-100 stock index price levels, 1998-2008

The GARCH-based volatility models

Let $r_t = 100(\ln C_t - \ln C_{t-1})$ is the daily return series, where C_t is the daily closing price on day *t*, and the set Ω_{t-1} contains the recorded returns up to time, t-1. The standard GARCH (1,1) specification is given below:

$$r_t = \mu + \varepsilon_t, \quad \varepsilon_t = \sigma_t z_t, \quad z_t|_{\Omega_{t-1}} \sim D(0,1) \qquad (1)$$

$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2 \qquad (2)$$

where μ and σ_t^2 are the conditional mean and variance of the r_t; ε_t is the innovation process; D(0,1) is a density function with a mean of zero; and $\omega \geq 0$, $\alpha \geq 0$ and $\beta \geq 0$. Furthermore, stationarity is achieved provided the $\alpha + \beta < 1$.

The two GARCH specifications which can cater for asymmetric volatility dynamics are, Exponential GARCH (EGARCH) proposed by Nelson (1991) and GJR-GARCH advocated by Glosten *et al.* (1993). With the same mean specification of that of the GARCH model, the conditional variance specification of the GJR-GARCH model is as below:

$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \psi I_{t-1}\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2 \qquad (3)$$

where I_{t-1} is the dummy variable which captures the asymmetric effect, such that $I_{t-1} = 1$ if $\varepsilon_{t-1} < 0$, and $I_{t-1} = 0$ if $\varepsilon_{t-1} \geq 0$. Thus, good news ($\varepsilon_{t-1} \geq 0$) has an impact of α, and bad news ($\varepsilon_{t-1} < 0$) has an impact of $\alpha + \psi$, the effect of bad news is even higher on conditional volatility if $\psi > 0$. Additionally, assuming the nonnegativity condition for the parameters ω, α and β with the restriction of $\alpha + \beta + 0.5\psi < 1$, whereas $\alpha + 0.5\psi$ should still be positive. With these parametric constraints the conditional variance specification of EGARCH model is as below:

$$\log(\sigma_t^2) = \omega + \alpha\left[v\frac{\varepsilon_{t-1}}{\sigma_{t-1}} + \frac{|\varepsilon_{t-1}|}{\sigma_{t-1}} - \sqrt{\frac{2}{\pi}}\right] + \beta\log(\sigma_{t-1}^2) \qquad (4)$$

In eq.(4): the parameter v gauges the asymmetric (leverage) component of information, the negative shocks of an equal magnitude of that of positive shock have higher impact if $v < 0$, significant α identify the volatility clustering effect. In particular, the logarithmic form of conditional variance conforms the nonnegativity of forecast variance irrespective of the estimated parameters are negative or positive.

Alternative errors distributions

From the seminal paper of Engle (1982), the z_t process in eq.(1) strictly follows a normal distribution. We assume that z_t follow SGT distribution, as is evident from the prior literature most of the time empirical returns series are skewed, leptokurtic and have fat-tails. As suggested by Theodossiou (1998), this study employs the following *SGT* distribution for z_t,

$$D(z_t; N, \kappa, \lambda) = C\left(1 + \frac{|z_t + \delta|^\kappa}{((N+1)/\kappa)(1 + \mathrm{Sign}(z_t + \delta)\lambda)^\kappa \theta^\kappa}\right)^{-(N+1)/\kappa} \qquad (5)$$

where

$$C = 0.5\kappa \cdot \left(\tfrac{N+1}{\kappa}\right)^{-1/\kappa} \cdot B\left(\tfrac{N}{\kappa}, \tfrac{1}{\kappa}\right)^{-1} \cdot \theta^{-1} \quad (6)$$

$$\theta = (g - \rho^2)^{-1/2} \quad (7)$$

$$\rho = 2\lambda \cdot B\left(\tfrac{N}{\kappa}, \tfrac{1}{\kappa}\right)^{-1} \cdot \left(\tfrac{N+1}{\kappa}\right)^{1/\kappa} \cdot B\left(\tfrac{N-1}{\kappa}, \tfrac{2}{\kappa}\right) \quad (8)$$

$$g = (1+3\lambda^2) \cdot B\left(\tfrac{N}{\kappa}, \tfrac{1}{\kappa}\right)^{-1} \cdot \left(\tfrac{N+1}{\kappa}\right)^{2/\kappa} \cdot B\left(\tfrac{N-2}{\kappa}, \tfrac{3}{\kappa}\right) \quad (9)$$

$$\delta = \rho \cdot \theta \quad (10)$$

where $z_t \sim D(0,1)$; the parameter N measures the tail-thickness with constraint N>2; the parameter κ observes the leptokurtosis with $\kappa > 0$; the parameter λ governs the skewness of the distribution with $|\lambda| < 1$; $B(\bullet)$ and Sign are the beta and sign function respectively. The *SGT* distribution reduces to numerous widely used distributions for the different combinations of parametric values. Particularly, for $\kappa = 2$ and $\lambda = 0$ it reduces to student-*t* distribution and for $N = \infty$, $\kappa = 2$ and $\lambda = 0$ it reduces to normal distribution. On the other hand recent articles, such as Politis (2004) and Hung et al. (2008) found some further evidences in the support of the heavy-tailed (HT) distributed errors. The HT distribution may better help to model the empirical distribution of asset returns series as they have fat-tails most of the time. This study also adopts the *HT* distributed errors for the innovation process, z_t, as follows:

$$D(z_t, a_0, 1) = \frac{(1+a_0 z_t^2)^{-1.5} \exp\left(-\frac{z_t^2}{2(1+a_0 z_t^2)}\right)}{\sqrt{2\pi}(\Phi(a_0^{-0.5}) - \Phi(-a_0^{-0.5}))} \quad (11)$$

where 1 is S.D of z_t, and Φ denotes the c.d.f of normal distribution. The shape parameter, a_0, reflects the degree of the heavy tails with constraint $0<a_0<1$. When a_0 approaches to zero, the *HT* will become standard normal distribution, whereas in contrast to normal distribution it has thicker tails as $a_0 \to 1$.

By placing various distributed errors (normal, student-*t*, *SGT* and *HT*) on the standard GARCH model, we then obtain four distribution-type volatility models (GARCH–*N*, –*T*, –*SGT*, and –*HT*), and thus compare their forecasting performance with that of two asymmetry-type models (GJR-GARCH and EGARCH models). Under such empirical design, we would ascertain the crucial factors for improving daily volatility forecasts of KSE-100 stock index between these two model categories.

The model parameter Θ is obtained through the use of the quasi maximum likelihood estimation (QMLE) method, as suggested in Bollerslev and Wooldridge (1992), maximizing the following sample log-likelihood function:

$$LL(\Theta) = \sum_{t=1}^{T} \log D(\Theta) \quad (12)$$

where $D(\Theta)$ is the likelihood function of the corresponding volatility model.

Volatility proxy measure and evaluation criteria

In order to evaluate the accuracy of daily volatility forecasts, we have to compare the model-based volatility forecasts with the true volatility, which is unobserved. Parkinson (1980) introduced a daily high-low range as a so-called PK volatility proxy, assuming the daily prices have a Brownian motion pattern. Due to the unavailability of high frequency data of KSE, this study uses the PK as a proxy for true volatility. The classical range-based estimator, PK is as given below:

$$\sigma_{PK,t}^2 = (4\ln 2)^{-1}(100 \times \ln(H_t / L_t))^2 \quad (13)$$

On a particular trading day t, H_t is the highest asset price and L_t is the lowest asset price.

The volatility forecast evaluation is performed using PK proxy in terms of MSE, MAE, MME(U), and MME(O), all of which are well-established within the literature and well-known. These forecasting error statistics are expressed as follows:

$$MSE = T^{-1} \sum_{t=1}^{T} (\sigma_{PK,t}^2 - \hat{\sigma}_t^2) \quad (14)$$

$$MAE = T^{-1} \sum_{t=1}^{T} \left|\sigma_{PK,t}^2 - \hat{\sigma}_t^2\right| \quad (15)$$

$$MME(U) = T^{-1} \sum_{t=1}^{T} UP_t, \quad \text{where} \quad UP_t = \begin{cases} |\sigma_{PK,t}^2 - \hat{\sigma}_t^2| & \text{if} \quad \sigma_{PK,t}^2 - \sigma_t^2 \le 0 \\ (\sigma_{PK,t}^2 - \hat{\sigma}_t^2)^{0.5} & \text{if} \quad 0 < \sigma_{PK,t}^2 - \sigma_t^2 \le 1 \\ (\sigma_{PK,t}^2 - \hat{\sigma}_t^2)^2 & \text{if} \quad \sigma_{PK,t}^2 - \sigma_t^2 > 1 \end{cases} \quad (16)$$

$$MME(O) = T^{-1} \sum_{t=1}^{T} OP_t, \quad \text{where} \quad OP_t = \begin{cases} |\sigma_{PK,t}^2 - \hat{\sigma}_t^2| & \text{if} \quad \sigma_{PK,t}^2 - \sigma_t^2 \ge 0 \\ |\sigma_{PK,t}^2 - \hat{\sigma}_t^2|^{0.5} & \text{if} \quad -1 \le \sigma_{PK,t}^2 - \sigma_t^2 < 0 \\ (\sigma_{PK,t}^2 - \hat{\sigma}_t^2)^2 & \text{if} \quad \sigma_{PK,t}^2 - \sigma_t^2 < -1 \end{cases} \quad (17)$$

where T denotes the number of forecast data points; $\sigma^2_{PK,t}$ signifies the PK volatility on day *t*; $\hat{\sigma}^2_t$ is the volatility forecast obtained from a model considered in this study for day *t*.

When a particular loss is smallest for a particular model, this does not guarantee its forecast superiority to a set of rival models. We employ the superior prediction ability (SPA) test proposed by Hansen (2005) to reveal statistical significance of a benchmark model relative to its various competitors.

Consider K+1 different models M_k for k = 0, 1, ..., K and which are discussed in previous section. For each model M_k, we generate T volatility forecasts $\hat{\sigma}^2_{k,t}$ for t = 1, 2, ..., T. Assuming that M_0 is the benchmark model, the loss function relative to the benchmark model is defined as:

$$f_{k,t} = L_{0,i,t} - L_{k,i,t}, \quad k = 1, 2, \ldots, K; t = 1, 2, \ldots, T \quad (18)$$

where i = MSE, MAE, MME(U) or MME(O). Assuming that models can be ranked consistently, then $\mu_k \equiv E[f_{k,t}]$ is well defined. When M_0 outperforms all other models, we have μ_k <0 for all models k = 1, 2, ..., K. Hence, the null hypothesis is that the benchmark model is not outperformed in terms of the specific loss function chosen:

$$\max_{k=1,\cdots,K} \mu_k \leq 0 \quad (19)$$

The corresponding SPA test statistic is given below:

$$SPA = \max_{k=1,\ldots,K} \frac{\sqrt{T} \cdot \bar{f}_k}{\hat{\omega}_{kk}} \quad (20)$$

with $\hat{\omega}^2_{kk}$ as a consistent estimate of ω^2_{kk}, and where $\bar{f}_k = T^{-1}\sum_{t=1}^{T} f_{k,t}$, $\omega^2_{kk} = \lim_{T\to\infty} var(\sqrt{T} \cdot \bar{f}_k)$. A consistent estimator of ω_{kk} and the *p*-value of test statistic, SPA, can be obtained via a bootstrap procedure proposed in Politis and Romano (1994).

Value-at-Risk application

To analyze the improving degree of model performance from risk management perspective, we conduct a reality check from a Value-at-Risk framework. According to Jorion (2000), the daily VaR of a GARCH-type model can be calculated as:

$$VaR_t(\alpha_0) = \mu + F(z_t;\alpha_0) \cdot \hat{\sigma}_t \quad (21)$$

where $F(z_t; \alpha_0)$ denotes the corresponding left quantile (α_0 = 0.5%, 1% or 5%) of an assumed distribution with specific parameters; and $\hat{\sigma}_t$ is the squared root of the daily conditional variance fore cast from a given model made at time *t*.

In analyzing the performance of the above volatility forecasting models for producing reasonable VaR estimates, the unconditional coverage test (LR_{UC}) of Kupiec (1995) is the most popular backtest among practitioners. Let $BL_t = I_{\{r_t < VaR_t(\alpha_0)\}}$, where $I(\cdot)$ is the usual indicator function. Given the backtest interval T, then n_1 ($= \sum_{t=1}^{T} BL_t$) is the number of the VaR violations (the number of days over a T period that the realized dollar loss was larger than the VaR estimate), while n_1/T is the violation frequency of that interval. Given the significance level α_0, the appropriate LR_{UC} statistic, under the null hypothesis that the expected violation frequency $n_1/T = \alpha_0$, equals

$$LR_{UC} = -2\ln\left(\alpha_0^{n_1}(1-\alpha_0)^{T-n_1}\right) + 2\ln\left((n_1/T)^{n_1}(1-n_1/T)^{T-n_1}\right) \quad (22)$$

which is asymptotically distributed as $\chi^2(1)$.

Note that the unconditional coverage test can reject a model having either too high or too low violation frequency, but has been criticized for its inability in response to serial correlation[2]. A dynamic quantile test (DQT) proposed by Engle and Manganelli (2004), this test considers not only the failure rate but also the serial correlation of the VaR violations. To perform such test, Engle and Manganelli define the hit sequence:

$$Hit_t = I_{\{r_t < VaR_t(\alpha_0)\}} - \alpha_0 \quad (23)$$

This sequence should be uncorrelated with past information and have a mean value of zero. To test for serial correlation in the hit sequence, Hit_t is regressed on five lags (days) and the current value of VaR. The DQT statistic is then computed as

$$DQT = \frac{\hat{\theta}'X'X\theta}{\alpha_0(1-\alpha_0)} \quad (24)$$

where $\hat{\theta}$ is the OLS estimates and X the vector of explanatory variables. The dynamic quantile test statistic, DQT, is asymptotically distributed as $\chi^2(7)$.

Results

Descriptive statistics of data

The basic descriptive of the KSE-100 stock index over the sample period under study are reported in Table 1. As is evident from Panel A of Table 1,

[2] If the VaR violations are apparently serially correlated, then there will be clustered loss exceeding the VaR which are likely to result in model risk.

the average daily returns over the sample period is positive while the standard deviation of the series is relatively higher. The returns series is characterized by skewness and kurtosis, skewed to the left as is evident from the significant negative value of skewness and has a high peak as is evident from the high kurtosis. Furthermore, the significant JB-statistic confirms that the returns series is not normally distributed. The significant; negative skewness, high kurtosis and Jarque-Bera statistic reveal that the returns series is no-normal and has thicker tails as compared to standard normal distribution. Moreover, the serial dependence is observed through the Ljung-Box Q statistic, while the LM-test reveals significant ARCH effects. The preliminary analysis of KSE-100 returns series prompt the use, of flexible distributions which cater for heavy tails (fat-tails) and of models which account for asymmetries. The results of PP (1998) and KPSS (1992) unit root tests of KSE-100 series are stated in the Panel B of Table 1, reported results do not support any evidence of non-stationarity in the returns series, specifying that no additional transformations are needed to model the KSE-100 return series. In the last, a significant Engle and Ng (1993) test statistic clearly reflects the asymmetric behavior of return's volatility and which necessitates the use of more flexible models which are capable to accommodate for leverage effects of volatility dynamics.

Table 1. Summary statistics of the daily KSE-100 returns from 1 January 1998 to 30 September 2008

Panel A. Descriptive statistics						
Mean %	S.D.	Skewness	Kurtosis	Jarque-Bera	$Q^2(24)$	LM(24)
0.063	1.788	-0.352 ^{c}	5.254 ^{c}	3059.587 ^{c}	1134.017 ^{c}	452.330 ^{c}
Panel B. Unit root tests						
PP	Bandwidth		KPSS		Bandwidth	
-45.100 ^{c}	13		0.185		7	
Panel C. Engle and Ng (1993)'s sign test for asymmetric dynamics in volatility						
Test statistic ($\sim\chi^2$ (3))			8.452 ^{b}			

Notes: 1. b and c denote significance at the 5% and 1% levels, respectively. 2. Jarque-Bera is the test statistic to test for the normality of a series. 3. $Q^2(24)$ is the Ljung-Box Q test for 24th order serial correlation of the squared returns. 4. LM test is applied to test for autocorrelation of the squared returns. 5. The bandwidth for the PP and KPSS test regressions are set using the Bartlett Kernel. A significant PP-test statistic rejects the null of non-stationarity and the critical values at 1%, 5% and 10% level of significance are -3.45, -2.864 and -2.568 respectively. A significant KPSS-test static rejects the null of stationarity and the critical values at 1%, 5% and 10% level of significance are 0.739, 0.463 and 0.347 respectively.

Estimation results

The estimation results[3] of the various models for the KSE-100 stock index during the in-sample period are reported in Table 2. First of all, Panel A of the Table 2 shows that the parameter estimates in the conditional variance equation are found to be statistically significant at the 1% level. In addition, the value of ($\alpha+\beta$) which varies from 0.901 to 0.978 is close to unity for each of the distribution-type models, indicating the presence of strong volatility persistence in the KSE-100 returns series. Second, the asymmetric parameter ψ of the GJR-GARCH model is positive and significant, while ν is negative and also statistically significant (both of ψ and ν are even significant at 1% level) in the EGARCH model, confirming that the KSE stock market exhibits a leverage effect with bad news exerting greater impact on KSE-100 returns series as compared to good news, even though the size (magnitude) of the shocks is the same. Third, the estimated shape parameters N, κ, λ, and a_0, are all highly significant and meet their parameters' constraints, reconfirming that the returns series of KSE-100 exhibits fat-tails and skewness and is leptokurtic. Finally, Panel B of the Table 2 reports the results of diagnostic tests. A comparison of log-likelihood function values (FV) for all competing models specifies the superior in-sample goodness of fit of GARCH-*SGT* model over the other mod-

[3] The parameters are estimated by QMLE and the BFGS optimization algorithm, using the econometric package of WinRATS 7.0.

els. In addition, $Q^2(24)$ computed on the squared standardized residuals for all models are insignificant. Such evidence necessitates the need of linear and non-linear GARCH specifications to purge away the effects of serial correlation in the conditional variance equation.

Table 2. Estimation results

Panel A. Model estimates						
Parameter	GARCH-*N*	GARCH-*t*	GARCH-*SGT*	GARCH-*HT*	GJR-GARCH	EGARCH
μ	0.046 [0.043]	0.078 [b] [0.039]	0.034 [0.043]	0.078 [b] [0.038]	0.019 [0.046]	0.002 [0.040]
ω	0.304 [c] [0.013]	0.207 [c] [0.026]	0.196 [c] [0.055]	0.126 [c] [0.034]	0.302 [c] [0.053]	0.060 [c] [0.003]
α	0.209 [c] [0.009]	0.213 [c] [0.016]	0.209 [c] [0.039]	0.137 [c] [0.025]	0.136 [c] [0.027]	0.251 [c] [0.012]
β	0.724 [c] [0.005]	0.761 [c] [0.010]	0.769 [c] [0.035]	0.764 [c] [0.035]	0.736 [c] [0.029]	0.958 [c] [0.002]
ψ	-	-	-	-	0.117 [c] [0.036]	-
ν	-	-	-	-	-	-0.138 [c] [0.038]
N	∞	4.409 [c] [0.371]	4.213 [c] [0.867]	-	∞	∞
κ	2	2	2.076 [c] [0.276]	-	2	2
λ	0	0	-0.082 [b] [0.038]	-	0	0
a_0	-	-	-	0.092 [c] [0.013]	-	-
Panel B. Diagnostic tests						
$Q^2(24)$	23.335	29.914	32.679	31.329	20.113	11.819
FV	-2377.142	-2305.839	-2303.821	-2305.920	-2372.637	-2356.461

Notes: 1. N, κ and λ are specific parameters of the SGT-distribution, where N and κ are positive kurtosis parameters controlling the tails and height of the density with N>2 and κ >0, respectively; λ denotes the skewness parameter obeying the constraint |λ|<1. Moreover, a0 denotes shape parameter of the HT-distribution governing the fat-tails of the densities with constraint 0<a0<1. 2. Standard errors are in brackets below corresponding parameter estimates. 3. a, b and c indicate significance at the 10%, 5% and 1% levels, respectively. 4. Q2(24) represents the Ljung-Box Q statistic of order 24 computed on the squared standardized residuals. 5. FV refers to the log-likelihood function value.

Analysis for volatility forecasting performance (2012-01-02)

To examine volatility forecasting performance, the forecast evaluation is performed using PK proxy measure based on both symmetric (MAE, MSE) and asymmetric (MME) loss criteria. In Table 3, column 2 reports the actual forecast error and column 3 reports the rank order of the included models. The MAE, MSE and MME(O) statistics all indicate that the GARCH-*HT* model yields the most accurate volatility forecasts, while EGARCH, GARCH-*N*, GJR-GARCH, GARCH-*SGT* models yield the volatility forecast from second to fifth place respectively. The GARCH-t model performs the worst, produces the largest/greatest forecasting error. If under-predictions are heavily penalized, the MME(U) statistic reveals that the GARCH-SGT and GARCH-t models provide the best and second best forecasts, respectively, while the remaining four models just perform marginally worse than the best model.

To further check the reliability of forecasting results and their statistical significance, the SPA test results are reported and listed in the last two columns of Table 3. The SPA test statistics (SPA_c and SPA_l) based on MAE, MSE and MME(O) all show that the GARCH-*HT* model is significantly superior to its competitors since it always produces higher consistent/liberal p-value than alternatives. As for the forecasting results obtained from the MME(U) criterion, the forecasting results are mixed. Specifically, when the GARCH-*HT* (EGARCH) is selected as the benchmark, the null hypothesis is significantly rejected, indicating that there exists a better model that outperforms the GARCH-*HT* (EGARCH) model.

Table 3. Out-of-sample volatility forecasting performance and SPA test results

Benchmark (M_0)	Value	Rank	SPA_c	SPA_l
Panel A. Model performance based on MAE				
GARCH-*N*	1.554	3	0.000	0.000
GARCH-*t*	1.717	6	0.000	0.000
GARCH-*HT*	1.209	1	0.495	0.495
GARCH-*SGT*	1.663	5	0.000	0.000
GJR-GARCH	1.571	4	0.000	0.000
EGARCH	1.534	2	0.000	0.000
Panel B. Model performance based on MSE				
GARCH-*N*	5.347	3	0.000	0.000
GARCH-*t*	7.249	6	0.000	0.000
GARCH-*H*T	4.061	1	0.452	0.452
GARCH-*SGT*	6.639	5	0.000	0.000
GJR-GARCH	5.366	4	0.000	0.000
EGARCH	4.753	2	0.001	0.001
Panel C. Model performance based on MME(O)				
GARCH-*N*	4.560	3	0.000	0.000
GARCH-*t*	6.674	6	0.000	0.000
GARCH-*HT*	2.902	1	0.444	0.444
GARCH-*SGT*	6.027	5	0.000	0.000
GJR-GARCH	4.641	4	0.000	0.000
EGARCH	3.921	2	0.000	0.000
Panel D. Model performance based on MME(U)				
GARCH-*N*	2.508	4	0.248	0.153
GARCH-*t*	2.462	2	0.202	0.159
GARCH-*HT*	2.611	6	0.089	0.089
GARCH-*SGT*	2.446	1	0.990	0.656
GJR-GARCH	2.464	3	0.754	0.362
EGARCH	2.534	5	0.100	0.066

Notes: 1. The true volatility is proxied by the daily high-low price range suggested by Parkinson (1980). 2. SPAc and SPAl denote the reality check p-values of the Hansen's consistent test and Hansen's liberal test, respectively. The null hypothesis is that none of the models is better than the benchmark. 3. The number of bootstrap replications to calculate the p-values is 1000 and the dependency parameter q is 0.5.

The descriptive statistics of the shape parameters N, κ, λ and a_0 for the KSE-100 return series during rolling period are shown in Table 4. Immediately observable from these statistics are that, each specific shape parameter for the rolling period meets its parameter constraint. On the one hand, the minimum and maximum values for the degree of freedom parameter N are 4.389 and 5.999, respectively. On the other hand, the parameters N, κ, and λ, which respectively range from 4.213 to 10.080, 1.753 to 2.294, and -0.214 and -0.053, indicate that the KSE-100 returns series is heavy-tailed, leptokurtic, and has a leftwards skew. In addition, as shown in Panel C of Table 4, the estimated value for the shape parameter, a_0, of HT distribution ranges between 0.069 and 0.096, indicating that the returns series displays evidence of fat-tails.

Table 4. Descriptive statistics for specified shape parameters in rolling window period

Parameter	Mean	S.D.	Min	Max
Panel A. *t*-distribution				
N	5.076	0.406	4.389	5.999
Panel B. *SGT*-distribution				
N	5.478	1.249	4.213	10.080
κ	2.031	0.118	1.753	2.294
λ	-0.127	0.045	-0.214	-0.053
Panel C. *HT*-distribution				
a_0	0.082	0.006	0.069	0.096

Notes: 1. N, κ and λ are specific parameters of the SGT-distribution, where N and κ are positive kurtosis parameters controlling the tails and height of the density with N>2 and κ >0, respectively; λ denotes the skewness parameter obeying the constraint $|\lambda|<1$. 2. a0 denotes shape parameter of the HT-distribution governing the fat-tails of the densities with constraint 0<a0<1.

Table 5. Out-of-sample VaR forecasting performance

Criterion	GARCH-N	GARCH-t	GARCH-HT	GARCH-SGT	GJR-GARCH	EGARCH
Panel A. 95% confidence level						
LR_{UC}	0.024	0.000	0.000	0.056	0.056	0.033
DQT	0.000	0.000	0.000	0.036	0.004	0.007
Panel B. 99% confidence level						
LR_{UC}	0.000	0.088	0.000	0.751	0.000	0.002
DQT	0.000	0.012	0.000	0.804	0.000	0.001
Panel C. 99.5% confidence level						
LR_{UC}	0.000	0.735	0.000	0.406	0.001	0.003
DQT	0.000	0.725	0.000	0.861	0.000	0.011

Notes: 1. This table shows asymptotic P-values for the unconditional coverage test (LRUC) and dynamic quantile test (DQT) statistics for the various VaR models under 95%, 99% and 99.5% confidence levels. 2. The LRUC and DQT statistics are asymptotically distributed $\chi^2(1)$ and $\chi^2(7)$, respectively. 3. The cells in boldface indicate rejection of the null hypothesis of correct VaR estimates at the 10% significance level.

Application to risk management practice (2012-01-02)

In this section, we employ the daily volatility forecasts obtained by the GARCH-*N*, GARCH-*t*, GARCH-*HT*, GARCH-*SGT*, GJR-GARCH, and EGARCH models to further examine their forecasting performance in the context of a VaR analysis. Table 5 presents the summary results of the out-of-sample VaR forecasts using unconditional coverage test (LR_{UC}) and dynamic quantile test (DQT) statistics under 95%, 99%, and 99.5% confidence levels.

First, we find that all models considered have been rejected by the LR_{UC} test at the 95% confidence level, indicating that each model has a statistically significant higher frequency of exceptions than allowed for at the 10% significant level. Second, the LR_{UC} test statistics in Panels B and C of Table 5 are all statistically significant, except for the GARCH-*SGT* at the 99% confidence level and the GARCH-*t* and GARCH-*SGT* at the 99.5% one. That is, the GARCH-*SGT* model can pass the unconditional coverage test at the 99% confidence level, while the empirical failure rate generated by either the GARCH-*t* or GARCH-*SGT* model is statistically consistent with the prescribed one at the 99.5% confidence level.

In addition to the unconditional coverage test, we employ the DQT test of Engle and Manganelli (2004) for further performance comparison. The DQT statistics in Panels A and B of Table 5 indicate that all models reject the null hypothesis of providing correct 5%, and 1% VaR estimates, except for the GARCH-*SGT* model at the 99% confidence level. As for the 99.5% confidence level situation, we find evidence that only the GARCH-*t* and GARCH-*SGT* models do not reject the null hypothesis of correct 0.5% VaR estimates.

From the previous results, it is quite evident that the GARCH model incorporated with the student-*t*- and *SGT*-distributed innovations are adequate in predicting daily VaRs of KSE-100 stock index under high confidence level.

Conclusions

In this article we empirically compare the daily volatility forecasting performance of distribution-type GARCH models with those of asymmetric-type ones for KSE-100 stock index over the period 1 January 1998 to 30 September 2008.

The descriptive statistics reveal that the returns series is negatively skewed, having fat tails with high kurtosis. Either the PP (1998) or KPSS (1992) test does not support the presence of unit root in series. In addition, the estimated conditional variance equations indicate the strong volatility persistence, while the asymmetric GJR-GARCH and EGARCH models specify the leverage effects in the returns series.

While examining the volatility performance of different models used in this study for both symmetric and asymmetric loss criteria, some important findings are observed. First, for MAE, MSE and MME(O) loss criteria the GARH-*HT* model yields the most accurate volatility forecasts. Second, for MME(U) loss criteria it is the GARCH-*SGT* which performs better than the other competing models. Moreover to check the robustness of the forecasting results, the SPA test results based on MAE, MSE and MME(O) all show that the GARCH-*HT* model is significantly superior to its competitors.

Finally, we apply the daily volatility forecasts generated by the various models to evaluate their VaR performance relating to KSE-100 returns as a reality check. The LR_{UC} and DQT test results under different confidence levels reveal that only the GARCH-*t* and GARCH-*SGT* are the appropriate models to predict the daily VaRs of KSE-100 stock index under high confidence level.

References

Andersen, T. G., & Bollerslev, T. (1997). Intraday periodicity and volatility persistence in financial markets. *Journal of Empirical Finance, 4*, 115–158.

Andersen, T. G., & Bollerslev, T. (1998). Answering the Skeptics: Yes, standard volatility models do provide accurate forecasts. *International Economic Review, 39(4)*, 885–905.

Awartani, B. M. A., & Corradi, V. (2005). Predicting the volatility of the S&P-500 stock index via GARCH models: The role of asymmetries. *International Journal of Forecasting, 21*, 167–183.

Bollerslev, T. (1986). Generalized autoregressive heteroskedasticity. *Journal of Econometrics, 31(3)*, 307–327.

Bollerslev, T., & Wooldridge, J. M. (1992). Quasi-maximum likelihood estimation and inference in dynamic models with time varying covariances. *Econometric Reviews, 11(2)*, 143–172.

Brailsford, T. J., & Faff, R. W. (1996). An evaluation of volatility forecasting techniques. *Journal of Banking and Finance, 20(3)*, 419–438.

Brooks C., & Persand, G. (2002). Model choice and Value-at-Risk performance. *Financial Analysts Journal, 58*, 87–97.

Chong, C.W., Ahmad, M.I., & Abdullah, M.Y. (1999). Performance of GARCH models in forecasting stock market volatility. *Journal of Forecasting, 18*, 333–343.

Chuang, I. Y., Lu, J. R., & Lee, P. H. (2007). Forecasting volatility in the financial markets: A comparison of alternative distributional assumptions. *Applied Financial Economics*, *17(13)*, 1051–1060.

Engle, R. F. (1982). Autoregressive conditional heteroscedasticity with estimates of the variance of U.K. inflation. *Econometrica, 50(4)*, 987–1008.

Engle, R. F., & Manganelli, S. (2004). CAViaR: Conditional autoregressive Value at Risk by regression quantiles. *Journal of Business and Economics Statistics, 22*, 367–381.

Engle, R. F., & Ng, V. K. (1993). Measuring and testing the impact of news on volatility. *Journal of Finance, 48*, 1749–1778.

Franses, P. H., & van Dijk, D. (1996). Forecasting stock market volatility using non-linear GARCH models. *Journal of Forecasting, 15(3)*, 229–235.

Glosten, L., Jagannathan, R., & Runkle, D. (1993). On the relation between the expected value and the volatility nominal excess return on stocks. *Journal of Finance, 46*, 1779–1801.

Gokcan, S. (2000). Forecasting volatility of emerging stock market: Linear versus non-linear GARCH models. *Journal of Forecasting, 19(6)*, 499–504.

Hansen, P. R. (2005). A test for superior predictive ability. *Journal of Business and Economic Statistics, 23(4)*, 365–380.

Heynen, R. C., & Kat, H. M. (1994). Volatility prediction: A comparison of stochastic volatility, GARCH(1,1) and EGARCH models. *Journal of Derivatives, 2*, 50–65.

Hung, J. C., Lee, M.C., & Liu, H. C. (2008). Estimation of Value-at-Risk for energy commodities via fat-tailed GARCH models. *Energy Economics, 30*, 1173–1191.

Jorion, P. (2000) Value at Risk: the new benchmark for managing financial risk. McGraw-Hill, New York.

Kupiec, P. (1995). Techniques for verifying the accuracy of risk measurement models. *Journal of Derivatives, 3(2)*, 73–84.

Kwiatkowski, D., Phillips, P. C. B., Schmidt, P., & Shin, Y. (1992). Testing the null hypothesis of stationarity against the alternative of a unit root: How sure are we that economic time series have a unit root? *Journal of Econometrics, 54(1-3),* 159–178.

Loudon, G. F., Watt, W. H., & Yadav, P. K. (2000). An empirical analysis of alternative parametric ARCH models. *Journal of Applied Econometrics, 15(2),* 117–136.

McMillan, D. G., & Speight, A. E. H. (2004). Daily volatility forecasts: Reassessing the performance of GARCH models. *Journal of Forecasting, 23(6)*, 449–460.

McMillan, D. G., Speight, A. E. H., & ap Gwilym, O. (2000). Forecasting UK stock market volatility. *Applied Financial Economics, 10(4)*, 435–448.

Nelson, D. B. (1991). Conditional heteroskedasticity in asset returns: A new approach. *Econometrica, 59(2)*, 347–370.

Parkinson, M. (1980). The extreme value method for estimating the variance of the rate of return. *Journal of Business, 53(1)*, 61–65.

Phillips, P. C. B., & Perron, P. (1988). Testing for a unit root in time series regression. *Biometrika, 75(2)*, 335–346.

Politis, D. N. (2004). A heavy-tailed distribution for arch residuals with application to volatility prediction. *Annals of Economics and Finance, 5*, 283–298.

Politis, D. N., & Romano, J. P. (1994). The stationary bootstrap. *Journal of the American Statistical Association, 89(428)*, 1303–1313.

Sadorsky, P. (2006). Modeling and forecasting petroleum futures volatility. *Energy Economics, 28*, 467–488.

Taylor, J. W. (2004). Volatility forecasting with smooth transition exponential smoothing. *International Journal of Forecasting, 20(2)*, 273–286.

Theodossiou, P. (1998). Financial data and the skewed generalized t distribution. *Management Science, 44(12)*, 1650–1661.

Wei, W. (2002). Forecasting stock market volatility with non-linear GARCH models: A case for china. *Applied Economics Letters, 9(3)*, 163–166.

Wilhelmsson, A. (2006). GARCH forecasting performance under different distribution assumptions. *Journal of Forecasting, 25*, 561–578.